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AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

Sixteenth General Meeting.

Boston, June 26, 27 and 28, 1899.

The opening session of the Sixteenth General Meeting was called to order in the Walker Building, Massachusetts Institute of Technology, on Monday, June 26, at 10.30 A. M., by President Kennelly.

The President:—I have great pleasure in introducing to the Institute His Honor, Mayor Quincy, who has kindly come to open our proceedings in the most acceptable and delightful manner.

Mayor Quincy:—Mr. President and Gentlemen of the American Institute of Electrical Engineers:—I have merely come here this morning to fulfil the pleasing and unusual duty of welcoming your association to Boston upon the occasion of its meeting here. Boston has been favored by being selected as the gathering place of a great many national organizations of this sort, and I know that our citizens generally appreciate the compliment thus paid us, and are anxious to do anything in their power to make the stay of such bodies in Boston pleasant, and to make Boston an agreeable place for such purposes.

The particular objects of your organization appeal to me as having the greatest interest for all who are concerned in the technical, industrial and financial progress of the country. I suppose that it would be impossible to find in industrial history an instance of such a rapid development and such large financial investment in a new line as has been afforded by the history of the development of electricity, particularly in this country, but also throughout the world, since the date of the formation of your Institute. That date seems to mark about the period when electrical development on various lines passed from its experimental or preliminary stage to the stage of an assured basis, to the stage of rapid growth and to the stage where it offered a field for the very large investment of capital, and the progress that has taken place since 1884 in the introduction, perfection and greater utilization of the telephone, of the arc light and of the incandescent light, and of the electric street railway and of electric motors for various purposes of power, seems to me to afford a history of unprecedented development, and I think that in the future those who study such things will look back upon this period of the last fifteen years as constituting one of the most interesting periods in the development of a new force, in the knowledge of how to handle it, and to apply it in a large way to the service of man, which can be found in the records of the world.

It has always seemed to me that the discovery which now seems to us so simple and elementary, of the possibility of converting power into electrical energy and transmitting it to great distances was, next to the invention of the steam engine, probably the greatest discovery that has ever been made; and as we look around us to-day and see the enormous factor which the telephone has become in the business life of the community, and is coming to fill more and more in the social life of the community, and perhaps most striking of all, as we see the enormous development of the electric railway, I think we can realize that we have to some extent already passed through and are passing through nothing less than an industrial revolution, a development which is so changing past methods and appliances that it is really entitled to that designation.

So much for the past. While the great inventions in electricity apparently have been made, while it is impossible it seems to me to conceive of such fundamental elementary discoveries in the field of electricity as have given us the electric motor, the telephone, the electric light and the electric street railway, yet I think we have by no means come to the end of electrical development upon the scientific and inventive side any more than we have upon the commercial side. While such a great change as has marked the introduction of electricity is scarcely possible in the future, yet I believe that the investment of capital in electricity will probably be still larger in the next fifteen years than it has been in the last fifteen years—enormous as that has been; and while the great main inventions, so to speak, have been made, I believe there is a vast work still to be done in the perfecting of various descriptions of apparatus and appliances for handling this comparatively new force.

It therefore seems to me a most excellent thing that the men who are making a study of this great subject from a strictly scientific and technical standpoint are joined together, organized in this AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS. The development of every line must be led by a comparatively small number of men who do the work of research, the work of scientific study and the work of development; and the electrical engineers of this country, comparatively few in number as they are, although somewhat rapidly growing in numbers within recent years, as our institutions of technical education are turning out new men to enter the ranks, have an opportunity to contribute in a very large way to the industrial and commercial development of this country; for of all forces I suppose that electricity is the most subtle; it is the most impalpable, its laws have been the most difficult to ascertain, and the field for scientific research in electricity certainly is still a very large and very fruitful one, and I believe that we are still to attain results of the very greatest importance through the theoretical study, through the scientific experiment and through the practical experience of such men as make up this organization.

Therefore it seems to me there can be no more interesting occasion than that which brings together the men who are engaged scientifically throughout the country in the development and in the practical handling of this great force of electricity; and I am sure that your deliberations and your exchange of experiences and of ideas and of scientific knowledge cannot but benefit this great line of thought and of development, and cannot but have a useful result for the many and varied interests which are now depending upon the continued development of this great new force which has come into the industrial life of the world, and which has already wrought such a marvelous revolution in the habits and methods of mankind.

The President:—I am sure we have all listened with great interest to the address of His Honor, and I feel sure that I am only voicing the general sentiment when I express to him the satisfaction we have had in listening to our opening address from him; and this meeting in Boston, the Sixteenth Meeting of this Institute, will be memorable to ourselves not merely from its having taken place in so great and historic a city, but having been called together under the auspices of such an excellent address of welcome. I have only one regret, and that is that His Honor is not one of us. It seems to me from the way he has handled some of the subjects he must have been one of us, and it is only by some oversight that he is not. We are much beholden to him, and we feel that he is in a sense one of our own workers.

The Secretary read a letter from Harvard University, also one from the Boston Electric Light Company; also one from the New England Telephone and Telegraph Company (offering for the use of members of the Institute to all parts of Boston and the suburbs a telephone set up in Room 14 of the building); also one from the American Telephone and Telegraph Company, the long distance service (offering for the use of the members a long distance telephone after six o'clock in the evening, set up in the Hotel Brunswick, and available to all points reached by the long distance telephone); one from the General Electric Company, Schenectady, New York, one from the Boston Terminal Company, one from Professor Elihu Thomson, extending an invitation to his house on Wednesday evening; one from the Vice-President's office of the Boston Elevated Railway; one from the Mayor of Cambridge; a letter of regret from Mr. Dunn saying he had been obliged to change his plans on account of his health, which necessitated his going to Haines Falls in the Catskills; a letter from the New England Gas and Coke Company, inviting the members to visit its new works at Everett.

THE PRESIDENT:—There is one report to be laid before the INSTITUTE this morning, and that is the report of the Committee on Standardization. The Chairman of that committee is Prof. Crocker, and I am sorry that Professor Crocker is abroad and not here to-day to present it. But this committee was appointed nearly two years ago and its provisional report was laid before the INSTITUTE at the last General Meeting a year ago at Omaha. The report has been before the INSTITUTE therefore for a year, and criticisms have been invited and a number have been received, and it is in printed form before you this morning. The matter is now before the INSTITUTE for action.

MR. A. V. GARRATT:—Mr. President, I believe that this report representing as it does a vast amount of careful, conscientious and well digested labor on the part of our very able Committee, should be either rejected or accepted as a whole as it is here printed before us. I therefore move you the acceptance of the report as presented to us in printed form and that it be published in the TRANSACTIONS.

[Motion seconded and adopted.]

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

HAVEMEYER BUILDING, NEW YORK, N. Y.
26 CORTLANDT STREET, JUNE, 1899.

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REPORT OF THE COMMITTEE ON STANDARDIZATION.

[Accepted by the INSTITUTE, June 26th, 1899.]

To The Council of The AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

Gentlemen :

Your committee on Standardization begs to submit the following report, covering such subjects as have been deemed of pressing and immediate importance, and which are of such a nature that general agreement may be expected upon them.

While it is the opinion of the committee that many other matters might advantageously have been considered, as, for example, standard methods of testing ; yet it has been deemed inexpedient to attempt to cover in a single report more than is here submitted.

Yours respectfully,

FRANCIS B. CROCKER, *Chairman.*
CARY T. HUTCHINSON,
A. E. KENNELLY.
J. W. LIEB, JR.
CHARLES P. STEINMETZ.
LEWIS B. STILLWELL.
ELIHU THOMSON.

GENERAL PLAN.

Efficiency. Sections 1 to 24

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(II)	Synchronous Machines,	“	10 to 11
(III)	Synchronous Commutating Machines,	“	12 to 15
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Appendices. (I) Efficiency.
(II) Apparent Efficiency.
(III) Power Factor and Inductance Factor.
(IV) Notation.
(V) Table of Sparking Distances.

Electrical Apparatus will be treated under the following heads:—

I. **Commutating Machines,** which comprise a constant magnetic field, a closed-coil armature, and a multi-segmental commutator connected thereto.

Under this head may be classed the following: Direct-current generators; direct-current motors; direct-current boosters; motor-generators; dynamotors; converters and closed-coil arc machines

A booster is a machine inserted in series in a circuit to change its voltage, and may be driven either by an electric motor, or otherwise. In the former case it is a motor-booster.

A motor-generator is a transforming device consisting of two machines; a motor and a generator, mechanically connected together.

A dynamotor is a transforming device combining both motor and generator action in one magnetic field, with two armatures or with an armature having two separate windings.

For Converters, see III.

II. **Synchronous Machines,** which comprise a constant magnetic field, and an armature receiving or delivering alternating currents in synchronism with the motion of the machine; *i. e.*, having a frequency equal to the product of the number of pairs of poles and the speed of the machine in revolutions per second.

III. **Synchronous Commutating Machines**:—These include: 1. Synchronous converters; *i. e.*, converters from alternating to direct, or from direct to alternating current, and 2. Double-current generators; *i. e.*, generators producing both direct and alternating currents.

A converter is a rotary device transforming electric energy from one form into another without passing it through the intermediary form of mechanical energy.

A converter may be either:

a. A direct current converter, converting from a direct current to a direct current or

b. A synchronous converter, formerly called a rotary converter, converting from an alternating to a direct current, or vice versa.

Phase converters, are converters from an alternating-current system to an alternating-current system of the same frequency but different phase.

Frequency converters are converters from an alternating-current system of one frequency to an alternating-current system of another frequency, with or without changes of phase.

IV. Rectifying Machines, or Pulsating-Current Generators, which produce a unidirectional current of periodically varying strength.

V. Stationary Induction Apparatus, *i. e.*, stationary apparatus changing electric energy from one form into another, without passing it through an intermediary form of energy. These comprise

a. Transformers, or stationary induction apparatus in which the primary and secondary windings are electrically insulated from each other.

b. Auto-transformers, formerly called compensators; *i. e.*, stationary induction apparatus in which part of the primary winding is used as a secondary winding; or conversely.

c. Potential regulators, or stationary induction apparatus having a coil in shunt, and a coil in series with the circuit, so arranged that the ratio of transformation between them is variable at will.

These may be divided into :—

1. Compensator potential regulators, in which the number of turns of one of the coils is changed
2. Induction potential-regulators, in which the relative positions of primary and secondary coils is changed.
3. Magneto potential-regulators, in which the direction of the magnetic flux with respect to the coils is changed.

d. Reactive coils, or Reactance coils, formerly called choking coils; *i. e.*, stationary induction apparatus used to produce impedance or phase displacement.

VI. Rotary Induction Apparatus, which consist of primary and secondary windings rotating with respect to each other. They comprise

a. Induction motors.

b. Induction generators.

c. Frequency changers.

d. Rotary phase converters.

EFFICIENCY.

1. The "efficiency" of an apparatus is the ratio of its net power output to its gross power input.[1]

2. Electric power should be measured at the terminals of the apparatus.

3. In determining the efficiency of alternating-current apparatus, the electric power should be measured when the current is in phase with the E. M. F., unless otherwise specified, except when a definite phase difference is inherent in the apparatus, as in induction motors, etc.

4. Mechanical power in machines should be measured at the pulley, gearing, coupling, etc , thus excluding the loss of power in said pulley, gearing or coupling, but including the bearing friction and windage. The magnitude of bearing friction and windage may be considered as independent of the load. The loss of power in the belt and the increase of bearing friction due to belt tension, should be excluded. Where, however, a machine is mounted upon the shaft of a prime mover, in such a manner that it cannot be separated therefrom, the frictional losses in bearings and in windage, which ought, by definition, to be included in determining the efficiency, should be excluded, owing to the practical impossibility of determining them satisfactorily. The brush friction, however, should be included.

a. Where a machine has auxiliary apparatus, such as an exciter, the power lost in the auxiliary apparatus should not be charged to the machine but to the plant consisting of machine and auxiliary apparatus taken together. The plant efficiency in such cases should be distinguished from the machine efficiency.

5. The efficiency may be determined by measuring all the losses individually and adding their sum to the output to derive the input, or subtracting their sum from the input to derive the output. All losses should be measured at, or reduced to, the temperature assumed in continuous operation, or in operation under conditions specified. (See Sections 25 to 31.)

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[1] An exception should be noted in the case of storage batteries or apparatus for storing energy, in which the efficiency, unless otherwise qualified, should be understood as the ratio of the energy output to the energy intake in a normal cycle.

In order to consider the application of the foregoing rules to various machines in general use, the latter may be conveniently divided into classes as follows:

I. Commutating Machines.

6. In commutating machines the losses are :—

a. Bearing friction and windage. (See Section 4.)

b Molecular magnetic friction, and eddy currents in iron and copper. These losses should be determined with the machine on open circuit, and at a voltage equal to the rated voltage $+Ir$ in a generator, and—Ir in a motor, where I denotes the current strength, and r denotes the internal resistance of the machine. They should be measured at the correct speed and voltage, since they do not usually vary in proportion to the speed or to any definite power of the voltage.

c. Armature resistance losses, I^2r', where I is the current strength in the armature, and r' is the resistance between armature brushes, excluding the resistance of brushes and brush contacts.

d. Comutator brush friction.

e. Commutator brush-contact resistance. It is desirable to point out that with carbon brushes the losses (*d*) and (*e*) are usually considerable in low-voltage machines.

f. Field excitation. With separately excited fields, the loss of power in the resistance of the field coils alone should be considered. With shunt fields or series fields, however, the loss of power in the accompanying rheostat should also be included, the said rheostat being considered as an essential part of the machine, and not as separate auxiliary apparatus.

(*b*) and (*c*) are losses in the armature or "armature losses;" (*d*) and (*e*) "commutator losses;" (*f*) "field losses."

7. The difference between the total losses under load and the sum of the losses above specified, should be considered as "load losses," and are usually trivial in commutating machines of small field distortion. When the field distortion is large, as is shown by the necessity for shifting the brushes between no load and full load, or with variations of load, these load losses may be considerable, and should be taken into account. In this case the efficiency may be determined either by input and output measurements, or the load losses may be estimated by the method of Section II.

8. Boosters should be considered and treated like other direct-current machines in regard to losses.

9. In motor-generators, dynamotors or converters, the efficiency is the

$$\frac{\text{electric output}}{\text{electric input}}.$$

II. Synchronous Machines.—

10. In synchronous machines the output or input should be measured with the current in phase with the terminal E. M. F., except when otherwise expressly specified.

Owing to the uncertainty necessarily involved in the approximation of load losses, it is preferable, whenever possible, to determine the efficiency of synchronous machines by input and output tests.

11. The losses in synchronous machines are ;

a. Bearing friction and windage ; see Sec. 4.

b. Molecular magnetic friction and eddy currents in iron, copper and other metallic parts. These losses should be determined at open circuit of the machine at the rated speed and at the rated voltage, $+ Ir$ in a synchronous generator, $- Ir$ in a synchronous motor, where $I =$ current in armature, $r =$ armature resistance. It is undesirable to compute these losses from observations made at other speeds or voltages.

These losses may be determined either by driving the machine by a motor, or by running it as a synchronous motor, and adjusting its fields so as to get minimum current input and measuring the input by wattmeter. The former is the preferable method, and in polyphase machines the latter method is liable to give erroneous results in consequence of unequal distribution of currents in the different circuits caused by inequalities of the impedance of connecting leads, etc.

c. Armature-resistance loss, which may be expressed by $p\ I^2\ r$; where r = resistance of one armature circuit or branch, I = the current in such armature circuit or branch, and p = the number of armature circuits or branches.

d. Load losses as defined in section 7. While these losses cannot well be determined individually, they may be considerable and, therefore, their joint influence should be determined by observation. This can be done by operating the machine on short circuit and at full-load current, that is, by determining what may be called the "short-circuit core loss." With the low field intensity and great lag of current existing in this case, the load losses are usually greatly exaggerated.

One-third of the short-circuit core loss may, as an approximation, and in the absence of more accurate information, be assumed as the load loss.

e. Collector-ring friction and contact resistance. These are generally negligible, except in machines of extremely low voltage.

f. Field excitation. In separately-excited machines, the $I^2\ r$ of the field coils proper should be used. In self-exciting machines, however, the loss in the field rheostat should be included. (See Section 6 *f*.)

III. Synchronous Commutating Machines.

12. In synchronous converters, the power on the alternating-current side is to be measured with the current in phase with the terminal E. M. F., unless otherwise specified.

13 In double-current generators, the efficiency of the machine should be determined as a direct-current generator in accordance with section 6., and as an alternating-current generator in accordance with section 11. The two values of efficiency may be different, and should be clearly distinguished.

14. In synchronous converters the losses should be determined when driving the machine by a motor. These losses are :—

a. Bearing friction and windage, see section 4.

b. Molecular magnetic friction and eddy currents in iron, copper and metallic parts. These losses should be determined at open circuit and at the rated terminal voltage, no allowance being made for the armature resistance, since the alternating and the direct currents flow in opposite directions.

c. Armature resistance. The loss in the armature is $q\ I^2\ r$, where I = direct current in armature, r = armature resistance and q, a factor which is equal to 1.37 in single phasers, 0.56 in three-phasers 0.37 in quarter-phasers and 0.26 in six-phasers

d Load losses The load losses should be determined in the same manner as described in section 11 *d*., with reference to the direct-current side.

e and *f*. Losses in commutator and collector friction and brush-contact resistance. See sections 6 and 11

g. Field excitation. In separately-excited fields, the $I^2\ r$ loss in the field coils proper should be taken, while in shunt and series fields the rheostat loss should be included, except where fields and rheostats are intentionally modified to produce effects outside of the conversion of electric power, as for producing phase displacement for voltage control. In this case 25 per cent. of the $I^2\ r$ loss in the field proper at non-inductive alternating circuit should be added as proper estimated allowance for normal rheostat losses. (See Section 6 *f*.)

15 Where two similar synchronous machines are available, their efficiency can be determined by operating one machine as a converter from direct to alternating, and the other as a converter from alternating to direct, connecting the alternating sides together, and measuring the difference between the direct-current input, and the direct-current output. This process may be modified by returning the output of the second machine through two boosters into the first machine and measuring the losses. Another modification might be to supply the losses by an alternator between the two machines, using potential regulators.

IV. Rectifying Machines or Pulsating-Current Generators.—

16. These include: Open-coil arc machines, constant-current rectifiers, constant-potential rectifiers.

The losses in open-coil arc machines are essentially the same as in sections 6 to 9 (closed-coil commutating machines.) In alternating-current rectifiers, however, the output must be measured by wattmeter and not by voltmeter and ammeter, since owing to the pulsation of current and E. M. F., a considerable discrepancy may exist between watts and volt-amperes, amounting to as much as 10 or 15 per cent.

17. In constant-current rectifiers, transforming from constant-potential alternating to constant direct current by means of constant-current transformers and rectifying commutators, the losses in the transformers are to be included in the efficiency and have to be measured when operating the rectifier, since in this case the losses are generally greater than when feeding an alternating secondary circuit. In constant-current transformers the load losses are usually larger than in constant-potential transformers and thus should not be neglected.

The most satisfactory method of determining the efficiency in rectifiers is to measure electric input and electric output by wattmeter. The input is usually not non-inductive, owing to a considerable phase displacement and to wave distortion. For this reason the apparent efficiency should also be considered, since it is usually much lower than the true efficiency. The power consumed by the synchronous motor or other source driving the rectifier should be included in the electric input.

V. Stationary Induction Apparatus.—

18. Since the efficiency of induction apparatus depends upon the wave shape of E. M. F., it should be referred to a sine wave of E. M. F., except where expressly specified otherwise. The efficiency should be measured with non-inductive load, and at rated frequency, except where expressly specified otherwise. The losses are:

a. Molecular magnetic friction and eddy currents measured at open circuit and at rated voltage — Ir, where I = rated current, r = resistance of primary circuit.

b. Resistance losses, the sum of the $I^2 r$ of primary and of secondary in a transformer, or of the two sections of the coil in the compensator or auto-transformer, where I = current in the coil or section of coil, r = resistance.

c. Load losses, *i. e.*, eddy currents in the iron and especially in the copper conductors, caused by the current. They should be measured by short-circuiting the secondary of the transformer and impressing upon the primary an E. M. F. sufficient to send full-load current through the transformer. The loss in the transformer under these conditions measured by wattmeter gives the load losses + $I^2 r$ losses in both primary and secondary coils.

d. Losses due to the methods of cooling, as power consumed by the blower in air-blast transformers, and power consumed by the motor driving pumps in oil or water-cooled transformers. Where the same cooling apparatus supplies a number of transformers or is installed to supply future additions, allowance should be made therefor.

19. In potential regulators the efficiency should be taken at the maximum voltage for which the apparatus is designed, and with non-inductive load, unless otherwise specified.

VI. Rotary Induction Apparatus.

20. Owing to the existence of load losses and since the magnetic density in the induction motor under load changes in a complex manner, the efficiency should be determined by measuring the electric input by wattmeter and the mechanical output at the pulley, gear, coupling, etc.

21. The efficiency should be determined at the rated frequency and the input measured with sine waves of impressed E. M. F.

22. The efficiency may be calculated from the apparent input, the power factor, and the power output. The same applies to induction generators. Since phase displacement is inherent in induction machines, their apparent efficiency is also important.

23. In frequency changers; *i. e.*, apparatus transforming from a polyphase system to an alternating system of different frequency, with or without a change in the number of phases, and phase converters; *i. e.*, apparatus converting from an alternating system, usually single phase, to another alternating system, usually polyphase, of the same frequency, the efficiency should also be determined by measuring both output and input.

VII. Transmission Lines.

24. The efficiency of transmission lines should be measured with non-inductive load at the receiving end, with the rated receiving pressure and frequency, also with sinusoidal impressed E. M. F.'s, except where expressly specified otherwise, and with the exclusion of transformers or other apparatus at the ends of the line.

RISE OF TEMPERATURE.

General Principles.—

25. Under regular service conditions, the temperature of electrical machinery should never be allowed to remain at a point at which permanent deterioration of its insulating material takes place.

26. The rise of temperature should be referred to the standard conditions of a room-temperature of 25° *C.*, a barometric pressure of 760 mm. and normal conditions of ventilation; that is, the apparatus under test should neither be exposed to draught, nor enclosed, except where expressly specified.

27. If the room temperature during the test differs from 25° *C.*, the observed rise of temperature should be corrected by 1/2 per cent for each degree *C.*[2] Thus with a room temperature of 35° *C.*, the observed rise of temperature has to be decreased by 5 per cent, and with a room temperature of 15° *C.*, the observed rise of temperature has to be increased by 5 per cent. The thermometer indicating the room temperature should be screened from thermal radiation emitted by heated bodies, or from draughts of air. When it is impracticable to secure normal conditions of ventilation on account of an adjacent engine, or other sources of heat, the thermometer for measuring the air temperature should be placed so as fairly to indicate the temperature which the machine would have if it were idle, in order that the rise of temperature determined shall be that caused by the operation of the machine.

28. The temperature should be measured after a run of sufficient duration to reach practical constancy. This is usually from 6 to 18 hours, according to the size and construction of the apparatus. It is permissible, however, to shorten the time of the test by running a lesser time on an overload in current and voltage, then reducing the load to normal, and maintaining it thus until the temperature has become constant.

In apparatus intended for intermittent service, as railway motors, starting rheostats, etc., the rise of temperature should be measured after a shorter time, depending upon the nature of the service, and should be specified.

In apparatus which by the nature of their service may be exposed to overload, as railway converters, and in very high voltage circuits, a smaller rise of temperature should be specified than in apparatus not liable to overloads or in low-voltage apparatus. In apparatus built for conditions of limited space, as railway motors, a higher rise of temperature must be allowed.

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[2] This correction is also intended to compensate, as nearly as is at present practicable, for the error involved in the assumption of a constant temperature coefficient of resistivity; *i. e.* 0.4 per cent. per deg. C. taken with varying initial temperatures.

29. In electrical conductors, the rise of temperature should be determined by their increase of resistance. For this purpose the resistance may be measured either by galvanometer test, or by drop-of-potential method. A temperature coefficient of 0.4 per cent per degree C., may be assumed for copper.[3] Temperature elevations measured in this way are usually in excess of temperature elevations measured by thermometers.

30. It is recommended that the following maximum values of temperature elevation should not be exceeded :

Commutating machines, rectifying machines, and synchronous machines.
  Field and armature, by resistance, 50° C.
  Commutator and collector rings and brushes, by thermometer, 55° C.
  Bearings and other parts of machine, by thermometer, 40° C.
Rotary induction apparatus:
  Electric circuits, 50° C., by resistance.
  Bearings and other parts of the machine 40° C., by thermometer.

In squirrel cage or short-circuited armatures, 55° C., by thermometer, may be allowed.

Transformers for continuous service—electric circuits by resistance 50° C., other parts by thermometer, 40° C., under conditions of normal ventilation.

Reactive coils, induction and magneto regulators—electric circuits by resistance 55°C., other parts by thermometer 45°C.

Where a thermometer, applied to a coil or winding, indicates a higher temperature elevation than that shown by resistance measurement, the thermometer indication should be accepted. In using the thermometer, care should be taken so to protect its bulb as to prevent radiation from it, and, at the same time, not to interfere seriously with the normal radiation from the part to which it is applied.

31. In the case of apparatus intended for intermittent service, the temperature elevation which is attained at the end of the period corresponding to the term of full load, should not exceed 50° C by resistance in electric circuits. In the case of transformers intended for intermittent service, or not operating continuously at full load, but continuously in circuit, as in the ordinary case of lighting transformers, the temperature elevation above the surrounding air temperature should not exceed 50° C by resistance in electric circuits and 40° C by thermometer in other parts, after the period corresponding to the term of full load. In this instance, the test load should not be applied until the transformer has been in circuit for a sufficient time to attain the temperature elevation due to core loss. With transformers for commercial lighting, the duration of the full-load test may be taken as three hours, unless otherwise specified. In the case of railway, crane, and elevator motors, the conditions of service are necessarily so varied that no specific period corresponding to the full-load term can be stated.

INSULATION.

32. The ohmic resistance of the insulation is of secondary importance only, as compared with the dielectric strength, or resistance to rupture by high voltage.

Since the ohmic resistance of the insulation can be very greatly increased by baking, but the dielectric strength is liable to be weakened thereby, it is preferable to specify a high dielectric strength rather than a high insulation resistance. The high voltage test for dielectric strength should always be applied.

Insulation Resistance.

33. Insulation resistance tests should, if possible, be made at the pressure for which the apparatus is designed.

The insulation resistance of the complete apparatus must be such that the rated voltage of the apparatus will not send more than $\frac{1}{1,000,000}$ of the full load current, at the rated terminal voltage, through the insulation. Where the value found in this way exceeds 1 megohm, 1 megohm is sufficient.

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[3] By the formula $R_T = R_t\,(1+0.004\,\theta)$. Where R_t is the resistance at room temperature, R_T the resistance when heated, and θ the temperature elevation $(T-t)$ in degrees centigrade.

Dielectric Strength.

34. The dielectric strength or resistance to rupture should be determined by a continued application of an alternating E. M. F. for one minute The source of alternating E. M. F. should be a transformer of such size that the charging current of the apparatus as a condenser does not exceed 25% of the rated capacity of the transformer.

35. The high voltage tests should not be applied when the insulation is low, owing to dirt or moisture, and should be applied before the machine is put into commercial service.

36. It should be pointed out that tests at high-voltages considerably in excess of the normal voltages are admissible on new machines, to determine whether they fulfill their specifications, but should not be made subsequently at a voltage much exceeding the normal, as the actual insulation of the machine may be weakened by such tests

37. The test for dielectric strength should be made with the completely assembled apparatus and not with its individual parts, and the voltage should be applied as follows ;—

1st. Between electric circuits and surrounding conducting material, and,

2nd. Between adjacent electric circuits, where such exist, as in transformers.

The tests should be made with a sine wave of E. M. F., or where this is not available, at a voltage giving the same striking distance between needle points in air, as a sine wave of the specified E M. F., except where expressly specified otherwise. As needles, new sewing needles should be used. It is recommended to shunt the apparatus during the test by a spark gap of needle points set for a voltage exceeding the required voltage by 10%.

A table of approximate sparking distances is given in Appendix V.

38. The following voltages are recommended for apparatus not including transmission lines or switchboards :

Rated Terminal Voltage.	Capacity.	Testing Voltage.
Not exceeding 400 volts............	Under 10 K. W.....	1000 volts.
" " "	10 K.W. and over	1500 "
400 and over, but less than 800 volts.	Under 10 K. W.....	1500 "
" " " "	10 K. W. and over .	2000 "
800 " " 1200 "	Any	3500 "
1200 " " 2500 "	Any..............	5000 "
2500 "	Any............	Double the normal rated voltages.
Synchronous motor fields and fields of converters started from the alternating current side............		5000 volts.

Alternator field circuits should be tested under a breakdown test voltage corresponding to the rated voltage of the exciter, and referred to an output equal to the output of the alternator; *i. e.*, the exciter should be rated for this test as having an output equal to that of the machine it excites.

Condensers should be tested at twice their rated voltage and at their rated frequency.

The values in the table above, are effective values, or square roots of mean square, reduced to a sine wave of E. M. F.

39. In testing insulation between different electric circuits as between primary and secondary of transformers, the testing voltage must be chosen corresponding to the high-voltage circuit.

40. In transformers of from 10,000 volts to 20,000 volts, it should be considered as sufficient to operate the transformer at twice its rated voltage, by connecting first the one, and then the other terminal of the high-voltage winding to the core and to the low-voltage winding. The test of dielectric resistance between the low voltage winding and the core should be in accordance with the recommendation in Section 38, for similar voltages and capacities.

41. When machines or apparatus are to be operated in series, so as to employ the sum of their separate E. M. F's, the voltage should be referred to this sum, except where the frames of the machines are separately insulated both from gronnd and from each other.

REGULATION.

42 The term regulation should have the same meaning as the term "inherent regulation," at present frequently used.

43 The regulation of an apparatus intended for the generation of constant potential, constant current, constant speed, etc., is to be measured by the maximum variation of potential current, speed, etc., occurring within the range from full load to no load, under such constant conditions of operation as give the required full-load values, the condition of full load being considered in all cases as the normal condition of operation.

44 The regulation of an apparatus intended for the generation of a potential, current, speed etc., varying in a definite manner between full load and no load, is to be measured by the maximum variation of potential current, speed, etc., from the satisfied condition, under such constant constant conditions of operation as give the required full-load values.

If the manner in which the variation in potential, current, speed, etc., between full load and no load is not specified, it should be assumed to be a simple linear relation; *i. e.* undergoing uniform variation between full load and no load.

The regulation of an apparatus may, therefore, differ according to its qualification for use. Thus the regulation of a compound-wound generator specified as a constant-potential generator will be different from that it possesses when specified as an over-compounded generator.

45. The regulation is given in percentage of the full-load value of potential, current, speed, etc., and the apparatus should be steadily operated during the test under the same conditions as at full load.

46. The regulation of generators is to be determined at constant speed; of alternating apparatus at constant impressed frequency.

47. The regulation of a generator-unit, consisting of a generator united with a prime mover, should be determined at constant conditions of the prime mover; *i. e.* constant steam pressure, head, etc. It would include the inherent speed variations of the prime mover. For this reason the regulation of a generator-unit is to be distinguished from the regulation of either the prime-mover, or of the generator contained in it, when taken separately.

48. In apparatus generating, transforming or transmitting alternating currents, regulation should be understood to refer to non-inductive load, that is to a load in which the current is in phase with the E. M. F. at the output side of the apparatus, except where expressly specified otherwise.

49. In alternating apparatus receiving electric power, regulation should refer to a sine wave of E. M. F., except where expressly specified otherwise.

50 In commutating machines, rectifying machines and synchronous machines, as direct-current generators and motors, alternating-current and polyphase generators, the regulation is to be determined under the following conditions:

a. At constant excitation in separately excited fields

b. With constant resistance in shunt field circuits and

c. With constant resistance shunting series fields; *i. e.* the field adjustment should remain constant, and should be so chosen as to give the required full-load voltage at full-load current.

51 In constant-potential machines, the regulation is the ratio of the maximum difference of terminal voltage from the rated full-load value (occurring within the range from full load to open circuit) to the full-load terminal voltage.

52. In constant-current machines, the regulation is the ratio of the maximum difference of current from the rated full-load value (occurring within the range from full load to short circuit), to the full-load current.

53. In constant-power machines, the regulation is the ratio of maximum difference of power from the rated full load value (occurring within the range of operation specified) to the rated power.

54. In over-compounded machines, the regulation is the ratio of the maximum difference in voltage from a straight line connecting the no-load and full-load values of terminal voltage as function of the current, to the full-load terminal voltage.

55. In constant-speed continuous-current motors, the regulation is the ratio of the maximum variation of speed from its full load value (occurring within the range from full-load to no-load) to the full-load speed.

56 In transformers, the regulation is the ratio of the rise of secondary terminal voltage from full-load to no-load, (at constant primary impressed terminal voltage) to the secondary terminal voltage.

57. In induction motors, the regulation is the ratio of the rise of speed from full-load to no-load, (at constant impressed voltage), to the full-load speed. The regulation of an induction motor is, therefore, not identical with the slip of the motor, which is the ratio of the drop in speed from synchronism, to the synchronous speed.

58. In converters, dynamotors, motor-generators, and frequency changers, the regulation is the ratio of the maximum difference of terminal voltage at the output side from the rated full-load voltage, (at constant impressed voltage and at constant frequency), to the full-load voltage on the output side.

59. In transmission lines, feeders, etc., the regulation is the ratio of maximum voltage difference at the receiving end, between no-load and full non-inductive load, to the full-load voltage at the receiving end, with constant voltage impressed upon the sending end.

60. In steam engines, the regulation is the ratio of the maximum variation of speed in passing from full-load to no load (at constant steam pressure at the throttle), to the full-load speed.

61. In a turbine or other water-motor, the regulation is the ratio of the maximum variation of speed from full-load to no-load (at constant head of water; *i. e.*, at constant difference of level between tail race and head race), to the full-load speed.

Variation and Pulsation.—

62. In prime movers which do not give an absolutely uniform rate of rotation or speed, as in steam engines, the "variation" is the maximum angular displacement in position of the revolving member expressed in degrees, from the position it would occupy with uniform rotation, and with one revolution as 360°; and the pulsation is the ratio of the maximum change of speed in an engine cycle to the average speed.

63. In alternators or alternating-current circuits in general, the variation is the maximum difference in phase of the generated wave of E. M. F. from a wave of absolutely constant frequency, expressed in degrees, and is due to the variation of the prime mover. The pulsation is the ratio of the maximum change of frequency during an engine cycle to the average frequency.

64. If n = number of poles, the variation of an alternator is $\frac{n}{2}$ times the variation of its prime mover if direct connected, and $\frac{n}{2}p$ times the variation of the prime mover if rigidly connected thereto in the velocity ratio p.

65. The pulsation of an alternating-current circuit is the same as the pulsation of the prime mover of its alternator.

RATING.

66. Both electrical and mechanical power should be expressed in kilowatts, except when otherwise specified. Alternating-current apparatus should be rated in kilowatts on the basis of non-inductive condition; *i. e.*, with the current in phase with the terminal voltage.

67. Thus the electric power generated by an alternating-current apparatus equals its rating only at non-inductive load, that is when the current is in phase with the terminal voltage.

68. Apparent power should be expressed in kilovolt-amperes as distinguished from real power in kilowatts.

69 If a power-factor other than 100% is specified, the rating should be expressed in kilovolt-amperes and power-factor, at full load.

70. The full-load current of an electric generator is that current which with the rated full-load terminal voltage gives the rated kilowatts, but in alternating-current apparatus only at non-inductive load.

71. Thus in machines in which the full load voltage differs from the no-load voltage, the full-load current should refer to the former.

If P = rating of an electric generator and E = full-load terminal voltage, the full-load current is:

$I = \frac{P}{E}$ in a continuous-current machine or single phase-alternator.

$I = \frac{P}{E\sqrt{3}}$ in a three-phase alternator.

$I = \frac{P}{2E}$ in a quarter-phase alternator.

72. Constant-current machines such as series arc-light generators, should be rated in kilowatts based on terminal volts and amperes at full load.

73. The rating of a fuse or circuit breaker should be the current-strength at which it will open the circuit, and not the working-current strength.

Classification of Voltages and Frequencies.

74. In direct-current, low-tension generators, the following average terminal voltages are in general use and are recommended:

125 volts. 250 volts. 550 volts.

75. In direct-current, and alternating-current, low-pressure circuits, the following average terminal voltages are in general use and are recommended:

110 volts, 220 volts.

In direct-current power circuits, for railway and other service, 500 volts may be considered as standard.

76. In alternating-current, high-pressure circuits at the receiving end, the following pressures are in general use, and are recommended:

1000 volts. 2000 volts. 3000 volts. 6000 volts.
10000 volts. 15000 volts. 20000 volts.

77. In alternating-current high-pressure generators, or generating systems the following terminal voltages are in general use and are recommended:

1150 volts 2300 volts 3450 volts

These pressures allow of a maximum drop in transmission of 15% of the pressure at the receiving end. If the drop required is greater than 15%, the generator should be considered as special.

78. In alternating-current circuits, the following approximate frequencies are recommended as desirable:

25 ~ or 30 ~ 40 ~ 60 ~ 120 ~[4]

These frequencies are already in extensive use and it is deemed advisable to adhere to them as closely as possible.

Overload Capacities.

79. All guarantees on heating, regulation, sparking, etc., should apply to the rated load, except where expressly specified otherwise, and in alternating-current apparatus to the current in phase with the terminal E. M. F., except where a phase displacement is inherent in the apparatus.

80. All apparatus should be able to carry a reasonable overload without self destruction by heating, sparking, mechanical weakness, etc. and with an increase of temperature elevation not exceeding 15° C, above those specified for full loads. See Secs. 25 to 31.

81. Overload guarantees should refer to normal conditions of operation regarding speed, frequency, voltage, etc., and to non-inductive conditions in alternating apparatus, except where a phase displacement is inherent in the apparatus.

82. The following overload capacities are recommended:

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[4] The frequency of 120~ may be considered as covering the already existing commercial frequencies between 120~ and 140~, and the frequency of 60 ~ as covering the already existing commercial frequencies between 60 ~ and 70 ~.

1st. In direct-current generators and alternating-current generators; 25% for one-half hour.

2d. In direct-current motors and synchronous motors, 25% for one-half hour, 50% for one minute; except in railway motors and other apparatus intended for intermittent service.

3d. Induction motors. 25% for one-half hour, 50% for one minute.

4th. Synchronous converters. 50% for one half hour.

5th. Transformers 25% for one-half hour. Except in transformers connected to apparatus for which a different overload is guaranteed, in which case the same guarantees shall apply for the transformers as for the apparatus connected thereto.

6th, Exciters of alternators and other synchronous machines, 10% more overload than is required for the excitation of the synchronous machine at its guaranteed overload, and for the same period of time.

APPENDIX I.

EFFICIENCY.

Efficiency of Phase-Displacing Apparatus.

In apparatus producing phase displacement as, for example, synchronous compensators, exciters of induction generators, reactive coils, condensers, polarization cells, etc., the efficiency should be understood to be the ratio of the volt-ampere activity to the volt-ampere activity plus power loss.

The efficiency may be calculated by determining the losses individually, adding to them the volt-ampere activity, and then dividing the volt-ampere activity by the sum.

1st. In synchronous compensators and exciters of induction generators, the determination of losses is the same as in other synchronous machines under Sections 10 and 11.

2nd. In reactive coils the losses are molecular friction, eddy losses, and I^2r loss. They should be measured by wattmeter. The efficiency of reactive coils should be determined with a sine wave of impressed E. M. F., except where expressly specified otherwise.

3rd In condensers, the losses are due to dielectric hysteresis and leakage and should be determined by wattmeter with a sine wave of E. M. F.

4th. In polarization cells, the losses are those due to electric resistivity and a loss in the electrolyte of the nature of chemical hysteresis and are usually very considerable. They depend upon the frequency, voltage and temperature, and should be determined with a sine wave of impressed E. M. F., except where expressly specified otherwise.

APPENDIX II.

Apparent Efficiency.

In apparatus in which a phase displacement is inherent to their operation, apparent efficiency should be understood as the ratio of net power output to volt-ampere input.

Such apparatus comprise induction motors, reactive synchronous converters, synchronous converters controlling the voltage of an alternating current system, self-exciting synchronous motors, potential regulators, and open magnetic circuit transformers, etc.

Since the apparent efficiency of apparatus generating electric power depends upon the power factor of the load, the apparent efficiency, unless otherwise specified, should be referred to a load power-factor of unity.

APPENDIX III.

Power Factor and Inductance Factor.

The power factor in alternating circuits or apparatus may be defined as the ratio of the electric power, in watts, to volt-amperes.

The inductance factor is to be considered as the ratio of wattless volt-amperes to total volt-amperes.

Thus, if p = power factor, q = inductance factor, then

$$p^2 + q^2 = 1$$

The power factor is the $\frac{\text{(energy component of current or E. M. F.)}}{\text{(total current or E. M. F.)}}$ and the inductance factor is the

$$\frac{\text{(wattless component of current or E. M. F.)}}{\text{(total current or E. M. F.)}} = \frac{\text{true power}}{\text{volt-amperes.}}$$

Since the power-factor of apparatus supplying electric power depends upon the power-factor of the load, the power-factor of the load should be considered as unity, unless otherwise specified.

APPENDIX IV.

The following notation is recommended :—
E, e, voltage, E. M. F., potential difference
I, i, current
P, power
Φ, magnetic flux
$\mathcal{B}$, magnetic density
R, r, resistance
X x, reactance
Z, z, impedance
L, l, inductance
C, c, capacity
Vector quantities when used should be denoted by capital italics.

APPENDIX V.

Table of Sparking Distances in Air between Opposed Sharp Needle-Points, for Various Effective Sinusoidal Voltages, in inches and in centimetres.

Kilovolts Sq. Root of Mean Square.	Distance. Inches	Cms.	Kilovolts Sq. Root of Mean Square.	Distance. Inches	Cms.
5	0.225	0.57	60	4.65	11.8
10	0.47	1.19	70	5.85	14.9
15	0.725	1.84	80	7.1	18.0
20	1.0	2.54	90	8.35	21.2
25	1.3	3.3	100	9.6	24.4
30	1.625	4.1	110	10.75	27.3
35	2.0	5.1	120	11.85	30.1
40	2.45	6.2	130	12.95	32.9
45	2.95	7.5	140	13 95	35.4
50	3.55	9.0	150	15.0	38.1

A paper presented at the Sixteenth General Meeting of the American Institute of Electrical Engineers, Boston, June 26th, 1899, President Kennelly in the Chair.

SYMBOLIC REPRESENTATION OF GENERAL ALTERNATING WAVES AND OF DOUBLE FREQUENCY VECTOR PRODUCTS.

BY CHARLES PROTEUS STEINMETZ.

PART I.

a. Graphically alternating currents and E. M. F.'s are usually represented by vectors. A vector is a quantity having length and direction. The length represents the intensity, the direction the phase of the alternating wave. The vectors generally issue from the center of co-ordinates.

In the topographical method, however, which is more convenient for complex networks, as interlinked polyphase circuits, the alternating wave is represented by the straight line between two points, these points representing the absolute values of potential (with regard to any reference point chosen as co-ordinate center) and their connection the difference of potential, in phase and intensity. Algebraically these vectors are represented by complex quantities. The impedance, admittance, etc., of the circuit is a complex quantity also, in symbolic denotation.

Thus current, E. M. F., impedance and admittance are related by multiplication and division of complex quantities similar as current, E. M. F., resistance and conductance are related by Ohm's law in direct current circuits.

In direct current circuits, power is the product of current into E. M. F. In alternating current circuits, if:

$$E = e^{I} + j\, e^{II}$$

$$I = i^{I} + j\, i^{II}$$

The product:

$$P_0 = E I = (e^1 i^1 - e^{11} i^{11}) + j (e^{11} i^1 + e^1 i^{11})$$

is not the power, that is multiplication and division which are correct in the inter-relation of current, E. M. F., impedance, do not give a correct result in the inter-relation of E. M. F., current, power. The reason is that E I are vectors of the same frequency, and Z a constant numerical factor which thus does not change the frequency.

The power P, however, is of double frequency compared with E and I, and thus cannot be represented by a vector in the same diagram with E and I.

$P_0 = E I$ is a quantity of the same frequency with E and I and thus cannot represent the power.

b. Since the power is a quantity of double frequency of E and I, and thus the phase angle ω in E and I corresponds to a phase angle $2\,\omega$ in the power, it is of interest to investigate the product E I formed by doubling the phase angle.

Algebraically it is:

$$P = E I = (e^1 + j\, e^{11}) (i^1 + j\, i^{11}) =$$

$$(e^1 i^1 + j^2 e^{11} i^{11}) + (j\, e^{11} i^{11} + e^1 j\, i^{11})$$

Since $j^2 = -1$, that is 180° rotation for E and I, for the double frequency vector, P, $j^2 = +1$, or 360° rotation, and

$$j \times 1 = j$$

$$1 \times j = -j$$

Hence, substituting these values, it is:

$$P = [E I] = (e^1 i^1 + e^{11} i^{11}) + j (e^{11} i^1 - e^1 i^{11})$$

The symbol $[E I]$ here denotes the transfer from the frequency of E and I to the double frequency of P.

The product: $P = [E I]$ consists of two components; the real component:

$$P^1 = [E I]^1 = (e^1 i^1 + e^{11} i^{11})$$

and the imaginary component:

$$j\, P^1 = j\, [E I]^1 = j\, (e^{11} i^1 - e^1 i^{11})$$

The component:

$$P^{1} = [E\,I]^{1} = (e^{1}\,i^{1} + e^{11}\,i^{11})$$

is the power of the circuit, $= \mathrm{E\ I} \cos (\mathrm{E\ I})$

The component:

$$P^{j} = [E\,I]^{j} = (e^{11}\,i^{1} - e^{1}\,i^{11})$$

is what may be called the "wattless power," or the powerless or quadrature volt-amperes of the circuit, $= \mathrm{E\ I} \sin (\mathrm{E\ I})$.

The real component will be distinguished by the index 1, the imaginary or wattless component by the index j.

By introducing this symbolism, the power of an alternating circuit can be represented in the same way as in the direct current circuit, as the symbolic product of current and E.M.F.

Just as the symbolic expression of current and E.M.F. as complex quantity does not only give the mere intensity, but also the phase:

$$E = e^{1} + j\,e^{11}$$

$$\mathrm{E} = \sqrt{{e^{1}}^{2} + {e^{11}}^{2}}$$

$$\tan \varphi = \frac{e^{11}}{e^{1}}$$

so the double frequency vector product $P = [E\,I]$ denotes more than the mere power, by giving with its two components $P^{1} = [E\,I]^{1}$ and $P^{j} = [E\,I]^{j}$ the true energy volt-amperes, and the wattless volt-amperes.

If:

$$E = e^{1} + j\,e^{11}$$

$$I = i^{1} + j\,i^{11}$$

thus:

$$\mathrm{E} = \sqrt{{e^{1}}^{2} + {e^{11}}^{2}}$$

$$\mathrm{I} = \sqrt{{i^{1}}^{2} + {i^{11}}^{2}}$$

it is:

$$P^1 = [E\,I]^1 = (e^1\,i^1 + e^{11}\,i^{11})$$

$$P^j = [E\,I]^j = (e^{11}\,i^1 - e^1\,i^{11})$$

and:

$$\begin{aligned} {P^1}^2 + {P^j}^2 &= {e^1}^2\,{i^1}^2 + {e^{11}}^2\,{i^{11}}^2 + {e^{11}}^2\,{i^1}^2 + {e^1}^2\,{i^{11}}^2 \\ &= ({e^1}^2 + {e^{11}}^2)\,({i^1}^2 + {i^{11}}^2) \\ &= (\mathrm{E\,I})^2 \\ &= Q^2 \end{aligned}$$

where Q = total volt-amperes of circuit.

That is:

The true power P^1 and the wattless power P^j are the two rectangular components of the total apparent power Q of the circuit.

Herefrom it follows:

In symbolic representation as double frequency vector products, powers can be combined and resolved by the parallelogram of vectors just as currents and E.M.F.'s in graphical or symbolic representation.

Hereby the graphical methods of treatment of alternating current phenomena are extended to include double frequency quantities, as power, torque, etc.

It is:

$$\frac{P^1}{Q} = p = \cos\omega = \text{power factor.}$$

$$\frac{P^j}{Q} = q = \sin\omega = \text{inductance factor}$$

of the circuit, and the general expression of power is:

$$\begin{aligned} P &= Q\,(p + j\,q) \\ &= Q\,(\cos\omega + j\sin\omega) \end{aligned}$$

(*c*.) The introduction of the double frequency vector product $P = [E\,I]$ brings us outside of the limits of algebra, however, and the commutative principle of algebra: $a \times b = b \times a$, does not apply any more, but it is:

$$[E\,I] \text{ unlike } [I\,E]$$

since :

$$[E\,I] = [E\,I]^1 + j\,[E\,I]^j$$

$$[I\,E] = [I\,E]^1 + j\,[I\,E]^j$$

$$= [E\,I]^1 - j\,[E\,I]^j$$

it is :

$$[E\,I]^1 = [I\,E]^1$$

$$[E\,I]^j = -\,[I\,E]^j$$

that is, the imaginary component reverses its sign by the interchange of factors.

The physical meaning hereof is : if the wattless power $[E\,I]^j$ is lagging with regard to E, it is leading with regard to I.

The wattless component of power is absent, or the total apparent power is true power, if :

$$[E\,I]^j = (e^{11}\,i^1 - e^1\,i^{11}) = 0.$$

that is :

$$\frac{e^{11}}{e^1} = \frac{i^{11}}{i^1}$$

or :

$$\tan(E) = \tan(I),$$

that is, E and I are in phase or in opposition.

The true power is absent, or the total apparent power wattless, if :

$$[E\,I]^1 = (e^1\,i^1 + e^{11}\,i^{11}) = 0$$

that is :

$$\frac{e^{11}}{e^1} = -\,\frac{i^1}{i^{11}}$$

or :

$$\tan E = -\cot I$$

that is, E and I are in quadrature.

The wattless power is lagging (with regard to E, or leading with regard to I) if :

$$[E\,I]^j > 0$$

and leading if:

$$[E\,I]^j < 0$$

The true power is negative, that is, power returns, if:

$$[E\,I]^1 < 0$$

It is:

$$[E, -I] = [-E\,I] = -[E\,I]$$

$$[-E, -I] = +[E\,I]$$

that is, when representing the power of a circuit or a part of a circuit, current and E.M.F. must be considered in their proper *relative* phases, but their phase relation with the remaining part of the circuit is immaterial.

(*d.*) If:

$$P_1 = [E_1\,I_1], \qquad P_2 = [E_2\,I_2] \ldots . P_n = [E_n\,I_n]$$

are the symbolic expressions of the power of the different parts of a circuit or network of circuits, the total power of the whole circuit or network of circuits is:

$$P = P_1 + P_2 + \ldots . + P_n$$

and it is:

$$P^1 = P_1^1 + P_2^1 + \ldots . + P_n^1$$

$$P^j = P_1^j + P_2^j + \ldots . + P_n^j$$

In other words the total power in symbolic expression (true as well as wattless) of a circuit or system is the sum of the powers of its individual components, in symbolic expression.

The first equation is obviously directly a result from the law of conservation of energy.

One result derived herefrom is for instance:

If in a generator supplying power to a system the current is out of phase with the E.M.F. so as to give the wattless power P^j, the current can be brought into phase with the generator E.M.F., or the load on the generator made non-inductive by inserting anywhere in the circuit an apparatus producing the wattless power $-P^j$, that is, compensation for wattless currents in a system takes place regardless of the location of the compensating device.

Obviously between the compensating device and the source of wattless currents to be compensated for, wattless currents will flow, and for this reason it may be advisable to bring the compensator as near as possible to the circuit to be compensated.

(*e.*) Like power, torque is a double frequency vector product also, of magnetism and M.M.F. or current, and thus can be treated in the same way.

In an induction motor, the torque is the product of the magnetic flux in one direction into the component of secondary induced current in phase with the magnetic flux in time, but in quadrature position therewith in space, times the number of turns of this current, or since the induced E.M.F. is in quadrature and proportional to the magnetic flux and the number of turns, the torque of the induction motor is the product of the induced E.M.F. into the component of secondary current in quadrature therewith in time and space, or the product of the induced current into the component of induced E.M.F. in quadrature therewith in time and space.

Thus, if:

$E = e^1 + j\, e^{11}$ = induced E.M.F. in one direction in space.

$I = i^1 + j\, i^{11}$ = secondary current in the quadrature direction in space,

the torque is:

$$T = [E\,I]^j = e^{11}\, i^1 - e^1\, i^{11}.$$

By this equation the torque is given in watts, the meaning being that $T = [E\,I]^j$ is the power which would be exerted by the torque at synchronous speed, or the torque in synchronous watts.

The torque proper is then:

$$\frac{T}{2\,\pi\,N\,p}$$

where:

p = number of pairs of poles of the motor.

Numerous instances of the the application of this are given in my previous paper on the single-phase induction motor.

As a further instance, we may consider the case of two polyphase induction motors in concatenation; that is, two equal induction motors in which the secondary of the first motor is closed by the primary of the second motor, the motors being mechanically connected so as to run at the same speed.

In this case, let:

N = frequency of main circuit,

s = slip of the first motor from synchronism.

the frequency induced in the secondary of the first motor and thus impressed upon the primary of the second motor is, $s\ N$.

The speed of the first motor is $(1 - s)\ N$, thus the slip of the second motor, or the frequency induced in its secondary is

$$s\ N - (1 - s)\ N = (2\ s - 1)\ N.$$

Let:

e = counter E.M.F. induced in the secondary of the second motor, reduced to full frequency.

$Z_0 = r_0 - j\ x_0$ = primary self-inductive impedance.

$Z_1 = r_1 - j\ x_1$ = secondary self-inductive impedance.

$Y = g + j\ b$ = primary exciting admittance of each motor all reduced to full frequency and to the primary by the ratio of turns.

It is then:

Second motor:

secondary induced E.M.F.:

$$e\ (2\ s - 1)$$

secondary current:

$$I_1 = \frac{e\ (2\ s - 1)}{r_1 - j\ (2\ s - 1)\ x_1} = e\ (a_1 + j\ a_2)$$

where:

$$a_1 = \frac{(2\ s - 1)\ r_1}{r_1^2 + (2\ s - 1)^2\ x_1^2} \quad a_2 = \frac{(2\ s - 1)^2\ x_1}{r_1^2 + (2\ s - 1)^2\ x_1^2}.$$

primary exciting current:

$$I_0 = e\ (g + j\ b)$$

thus, total primary current:

$$I_2 = I_1 + I_0 = e\ (b_1 + j\ b_2)$$

where:

$$b_1 = a_1 + g \qquad\qquad b_2 = a_2 + b$$

primary induced E.M.F.:

$$s\,e$$

primary impedance voltage:

$$I_2\,(r_0 - j\,s\,x_0)$$

thus, primary impressed E.M.F.:

$$E_2 = s\,e + I_2\,(r_0 - j\,s\,x_0) = e\,(c_1 + j\,c_2)$$

where:

$$c_1 = s + r_0\,b_1 + s\,x_0\,b_2 \qquad\qquad c_2 = r_0\,b_2 - s\,x_0\,b_1$$

First motor:
secondary current:

$$I_2 = e\,(b_1 + j\,b_2)$$

secondary induced E.M.F.:

$$E_3 = E_2 + I_2\,(r_1 - j\,s\,x_1) = e\,(d_1 + j\,d_2)$$

where:

$$d_1 = c_1 + r_1\,b_1 + s\,x_1\,b_2 \qquad\qquad d_2 = c_2 + r_1\,b_2 - s\,x_1\,b_1$$

primary induced E.M.F.:

$$E_4 = \frac{E_3}{s} = e\,(f_1 + j\,f_2)$$

where:

$$f_1 = \frac{d_1}{s} \qquad\qquad f_2 = \frac{d_2}{s}\,*$$

primary exciting current:

$$I_4 = E_4\,(g + j\,b)$$

total primary current:

$$I = I_2 + I_4 = e\,(g_1 + j\,g_2)$$

---

* At $s = 0$ these terms f_1 and f_2 become indefinite, and thus at and very near synchronism have to be derived by substituting the complete expressions for f_1 and f_2.

where:

$$g_1 = b_1 + g\,f_1 - b\,f_2 \qquad g_2 = b_2 + g\,f_2 + b\,f_1$$

primary impedance voltage:

$$I\,(r_0 - j\,x_0)$$

thus, primary impressed E.M.F.:

$$E_0 = E_4 + I\,(r_0 - j\,x_0) = e\,(h_1 + j\,h_2)$$

where:

$$h_1 = f_1 + r_0\,g_1 + x_0\,g_2 \qquad h_2 = f_2 + r_0\,g_2 - x_0\,g_1$$

or, absolute:

$$e_0 = e\,\sqrt{h_1^2 + h_2^2}$$

and:

$$e = \frac{e_0}{\sqrt{h_1^2 + h_2^2}}$$

Substituting now this value of e in the preceding gives the values of the currents and E.M.F.'s in the different circuits of the motor series.

In the polyphase induction motor, the induced E.M.F. in quadrature in space to the induced E.M.F., E is:

$$j\,E.$$

Thus, in the second motor, the torque is:

$$T_2 = [\,j\,e\,I_1]^j = [e\,I_1]^1 = e^2\,a_1$$

hence, its power output:

$$\mathrm{P}_3 = (1 - s)\;T_2 = (1 - s)\,e^2\,a_1$$

The power input is:

$$P_2 = [E_2\,I_2] = [E_2\,I_2]^1 + j\,[E_2\,I_2]^j$$

$$= e^2\,[(c_1 + j\,c_2)\,(b_1 + j\,b_2)]$$

hence, the efficiency:

$$\frac{\mathrm{P}_3}{P_2^1} = \frac{(1 - s)\,e^2\,a_1}{[E_2\,I_2]^1} = \frac{(1 - s)\,a_1}{c_1\,b_1 + c_2\,b_2}$$

the power factor:

$$\frac{P_2^1}{Q} = \frac{[E_2\ I_2]^1}{\mathrm{E}_2\ \mathrm{I}_2} = \frac{c_1\, b_1 + c_2\, b_2}{\sqrt{(c_1^2 + c_2^2)\ (b_1^2 + b_2^2)}}$$

etc.

In the first motor:
the torque is:

$$T_1 = [E_4\ I_2]^1 = e^2\ [(f_1 + j\, f_2)\ (b_1 + j\ b_2)]^1$$

$$= e^2\ (f_1\, b_1 + f_2\, b_2)$$

the power output:

$$\mathrm{P}_4 = T_1\ (1 - s)$$

$$= e^2\ (1 - s)\ (f_1\, b_1 + f_2\, b_2)$$

the power input:

$$P_1 = [E_0\ I\,] = e^2\ [(h_1 + j\ h_2)\ (g_1 + j\ g_2)]$$

$$= [E_0\ I\,]^1 + j\ [E_0\ I\,]^j$$

Thus, the efficiency:

$$\frac{\mathrm{P}_4}{[E_0\ I\,]^1 - [E_2\ \ I_2]^1} = \frac{(1 - \mathrm{s})\ (f_1\, b_1 + f_2\, b_2)}{(h_1\, g_1 + h_2\, g_2) - (c_1\, b_1 + c_2\, b_2)}$$

the power factor of the whole system:

$$\frac{\mathrm{P}_1}{\mathrm{E}_0\,\mathrm{I}} = \frac{h_1\, g_1 + b_2\, g_2}{\sqrt{(h_1^2 + h_2^2)\ (g_1^2 + g_2^2)}}$$

the power factor of the first motor:

$$\frac{\mathrm{P}_1 - \mathrm{P}_2}{\mathrm{E}_0\,\mathrm{I} - \mathrm{E}_2\ \mathrm{I}_2} = \frac{(h_1\, g_1 + h_2\, g_2) - (c_1\, b_1 + c_2\, b_2)}{\sqrt{(h_1^2 + h_2^2)\,(g_1^2 + g_2^2)} - \sqrt{c_1^2 + c_2^2)\,(b_1^2 + b_2^2)}}$$

the total efficiency of the system:

$$\frac{\mathrm{P}_1 + \mathrm{P}_3}{[E_0\ \ I\,]^1} = \frac{(1 - s)\ (f_1 + b_1 + f_2\, b_2 + a_1)}{h_2\, g_1 + h_2\, g_2}$$

etc.

As instance are given in Fig. 1. the curves of total torque, of torque of the second motor, and of current, for the range of slip from $s = + 1.5$ to $s = - .7$ for a pair of induction motors in concatenation, of the constants:

$$Z_0 = Z_1 = .1 - .3\,j$$

$$Y = .01 + .1\,j.$$

Fig. 2 gives the curves of total torque and of current, from test of a motor of similar constants, for the range from

$$s = + 1 \text{ to } s = 0.$$

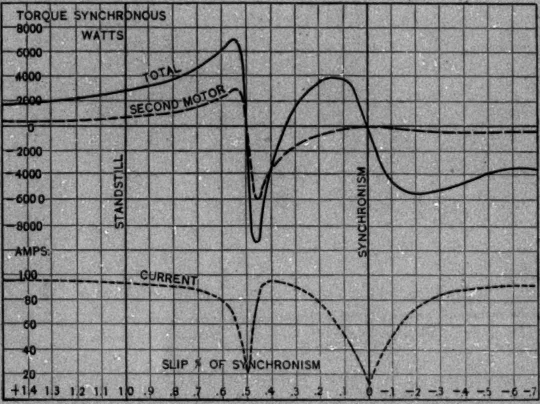

Fig. 1. Concatenation of Induction Motors. Speed Curves.

$Z = .1 - 8j \quad Y = .01 + .1j$

As seen, there are two ranges of positive torque for the whole system, one below half synchronism, and one from about ⅔ to full synchronism, and two ranges of negative torque, or generator action of the motor, from half to two-third synchronism, and above full synchronism.

With higher resistance in the secondary of the second motor, the second range of positive torque of the system disappears more or less and the torque curves become as shown in Fig. 3.

Part II.

(*a*.) The vector representation:

$$A = a^{1} + j\, a^{11} = a\,(\cos a + j \sin a)$$

of the alternating wave:

$$A = a_0 \cos (\varphi - a)$$

applies to the sine wave only.

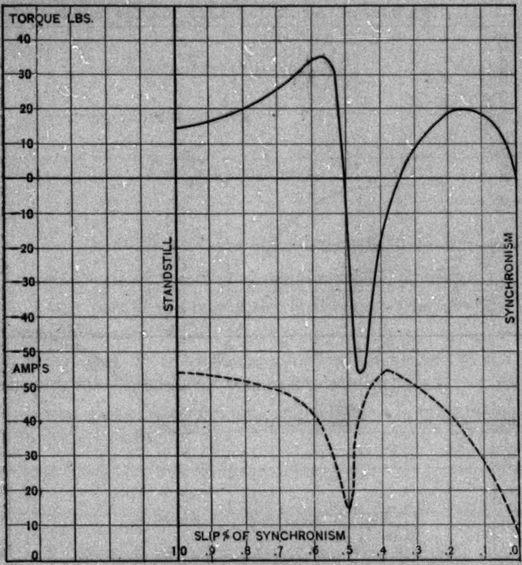

Fig. 2. Concatenation of Induction Motor. Speed Curves of two $I - 6 - 5 - 1200 - 110$.

The general alternating wave, however, contains an infinite series of terms, of odd frequencies:

$$A = A_1 \cos (\varphi - a_1) + A_3 \cos (3\, \varphi - a_3) + A_5 \cos (5\, \varphi - a_5) + .$$

thus can not directly be represented by one complex imaginary vector quantity.

The replacement of the general wave by its equivalent sine wave, that is a sine wave of equal effective intensity and equal power, while sufficiently accurate in many cases, completely fails in other cases, especially in circuits containing capacity, or in circuits containing periodically (and in synchronism with the wave) varying resistance or reactance (as alternating arcs, reaction machines, synchronous induction motors, oversaturated magnetic circuits etc.)

Since however, the individual harmonics of the general alternating wave are independent of each other, that is, all products of different harmonics vanish, each term can be represented by a complex symbol, and the equations of the general wave then are the resultants of those of the individual harmonics.

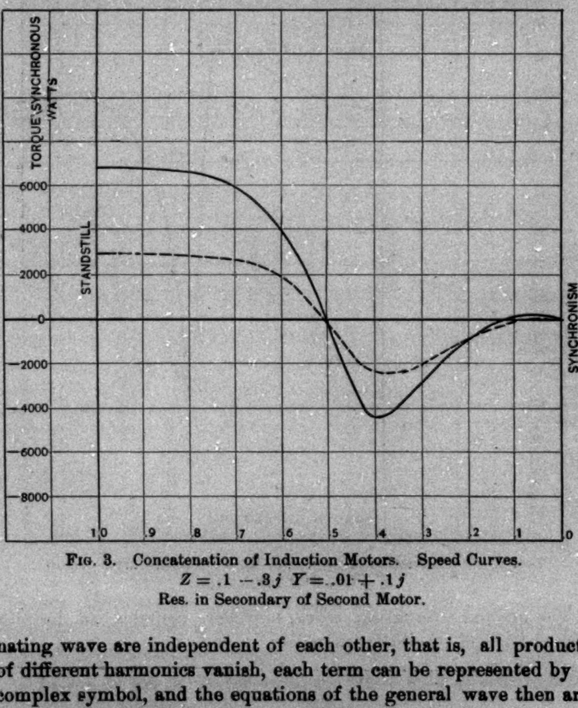

Fig. 3. Concatenation of Induction Motors. Speed Curves.
$Z = .1 - .3j \quad Y = .01 + .1j$
Res. in Secondary of Second Motor.

This can be represented symbolically, by combining in one formula symbolic representations of different frequencies, thus :

$$A = \sum_{1}^{\infty} {}^{2n-1} \left(a_n{}^{1} + j_n \; a_n{}^{11} \right)$$

where:

$$j_n = \sqrt{-1}$$

and the index of the j_n merely denotes that the j's of different indices n, while algebraically identical, physically represent different frequencies, and thus can not be combined.

The general wave of E.M.F. is thus represented by:

$$E = \sum_{1}^{\infty} {}^{2n-1} \left(e_n{}^{1} + j_n \, e_n{}^{11} \right)$$

the general wave of current by:

$$I = \sum_{1}^{\infty} {}^{2n-1} \left(i_n{}^{1} + j_n \, i_n{}^{11} \right)$$

If:

$$Z_1 = r - j \left(x_m + x_0 + x_c \right)$$

is the impedance of the fundamental harmonic, where:

x_m is that part of the reactance which is proportional to the frequency (inductance, etc.)

x_0 is that part of the reactance which is independent of the frequency (mutual induction, synchronous motion, etc.)

x_c is that part of the reactance which is inverse proportional to the frequency (capacity, etc).

the impedance for the nth harmonic is:

$$Z = r - j_n \left(n \, x_m + x_0 + \frac{x_c}{n} \right)$$

This term can be considered as the general symbolic expression of the impedance of a circuit of general wave shape.

Ohm's law, in symbolic expression, assumes for the general alternating wave the form:

$I = \frac{E}{Z}$ or:

$$\sum_{1}^{\infty} {}^{2n-1} \left(i_n{}^{1} + j_n i_n{}^{11} \right) = \sum_{1}^{\infty} {}^{2n-1} \frac{e_n{}^{1} + j_n e_n{}^{11}}{r - j_n \left(n x_m + x_0 + \frac{x_c}{n} \right)}$$

$E = I\,Z$ or:

$$\sum_{1}^{\infty}{}_{2n-1}\,(e_n{}^1 + j_n\,e_n{}^{11}) = \sum_{1}^{\infty}{}_{2n-1}\left[r - j_n\left(n x_m + x_0 + \frac{x_c}{n}\right)\right](i_n{}^1 + j_n i_n{}^{11})$$

$Z = \dfrac{E}{I}$ or:

$$Z = r - j_n\left(n\ x_m + x_0 + \frac{x_c}{n}\right) = \frac{e_n + j_n\,e_n{}^{11}}{i_n{}^1 + j_n\,i_n{}^{11}}$$

The symbols of multiplication and division of the terms E, I, Z, thus represent not algebraic operation, but multiplication and division of corresponding terms of E, I, Z, that is terms of the same index n, or, in algebraic multiplication and division of the series E, I, all compound terms, that is terms containing two different n, vanish.

(*b*.) The effective value of the general wave:

$$a = A_1 \cos(\varphi - a_1) + A_3 \cos(3\ \varphi - a_3) + A_5 \cos(5\ \varphi - a_5) + \ ..$$

is the square root of the sum of mean squares of individual harmonics:

$$\mathrm{A} = \sqrt{\tfrac{1}{2}\left\{A_1^2 + A_3^2 + A_5^2 + \ \dots\ \right\}}$$

since, as discussed above, the compound terms, of two different indices n, vanish, the absolute value of the general alternating wave:

$$A = \sum_{1}^{\infty}{}_{2n-1}\ \frac{a_n{}^1 + j_n\,a_n{}^{11}}{b_n{}^1 + j_n\,b_n{}^{11}}$$

is thus:

$$\mathrm{A} = \sqrt{\sum_{1}^{\infty}{}_{2n-1}\frac{a_n{}^{1\,2} + a_n{}^{11\,2}}{b_n{}^{1\,2} + b_n{}^{11\,2}}}$$

which offers an easy means of reduction from symbolic to absolute values.

Thus, the absolute value of the E.M.F.

$$E = \sum_1^\infty {}_{2n-1} (e_n^1 + j_n e_n^{11})$$

is:

$$\text{E} = \sqrt{\sum_1^\infty {}_{2n-1} (e_n^{1\,2} + e_n^{11\,2})}$$

the absolute value of the current:

$$I = \sum_1^\infty {}_{2n-1} (i_n^1 + j_n i_n^{11})$$

is:

$$\text{I} = \sqrt{\sum_1^\infty {}_{2n-1} (i_n^{1\,2} + i_n^{11\,2})}$$

(*c.*) The double frequency power (torque, etc.) equation of the general alternating wave has the same symbolic expression as with the sine wave:

$$\begin{aligned} P &= [E I] \\ &= P^1 + j P^j \\ &= [E I]^1 + j [E I]^j \\ &= \sum_1^\infty {}_{2n-1} (e_n^1 i_n^1 + e_n^{11} i_n^{11}) + \sum_1^\infty {}_{2n-1} j_n (e_n^{11} i_n^1 - e_n i_n^{11}) \end{aligned}$$

where:

$$P^1 = [E I]^1 = \sum_1^\infty {}_{2n-1} (e_n^1 i_n^1 + e_n^{11} i_n^{11})$$

$$P^j = [E I]^j = \sum_1^\infty {}_{2n-1} \frac{j_n}{j} (e_n^{11} i_n^1 - e_n^1 i_n^{11})$$

The j_n enters under the summation sign of the "wattless power P^j, so that the wattless powers of the different harmonics can not be algebraically added.

Thus:

The total "true power" of a general alternating current circuit is the algebraic sum of the powers of the individual harmonics.

The total "wattless power" of a general alternating current circuit is not the algebraic, but the absolute sum of the wattless powers of the individual harmonics.

Thus, regarding the wattless power as a whole, in the general alternating circuit no distinction can be made between lead and lag, since some harmonics may be leading, others lagging.

The apparent power, or total volt-amperes, of the circuit is:

$$Q = \mathrm{E\,I} = \sqrt{\sum_{1}^{\infty}{}_{2n-1}\left({e_n^{1}}^{2}+{e_n^{11}}^{2}\right)\sum_{1}^{\infty}{}_{2n-1}\left({i_n^{1}}^{2}+{i_n^{11}}^{2}\right)}$$

The power factor of the circuit is:

$$p = \frac{P^1}{Q} = \frac{\sum_{1}^{\infty}{}_{2n-1}\left(e_n^{1}\,i_n^{1} + e_n^{11}\,i_n^{11}\right)}{\sqrt{\sum_{1}^{\infty}{}_{2n-1}\left({e_n^{1}}^{2}+{e_n^{11}}^{2}\right)\sum_{1}^{\infty}{}_{2n-1}\left({i_n^{1}}^{2}+{i_n^{11}}^{2}\right)}}.$$

The term "inductance factor" however, has no meaning any more, since the wattless powers of the different harmonics are not directly comparable.

The quantity:

$$q_0 = \sqrt{1-p^2}$$

has no physical significance, and is not $\frac{\text{wattless power}}{\text{total apparent power}}$

The term:

$$\frac{P^1}{\mathrm{E\,I}}$$

$$= \sum_{1}^{\infty}{}_{2n-1}\,\frac{e_n^{11}\,i_n^{1} - e_n^{1}\,i_n^{11}}{\mathrm{E\,I}}$$

$$= \sum_{1}^{\infty}{}_{2n-1}\,q_n$$

where:

$$q_n = \frac{e_n^{11}\,i_n^{1} - e_n^{1}\,i_n^{11}}{\mathrm{E\,I}}$$

consists of a series of inductance factors q_n of the individual harmonics.

As a rule,

$$p^2 + q^2 < 1$$

for the general alternating wave, that is q differs from

$$q_0 = \sqrt{1 - p^2}$$

The complex quantity:

$$U = \frac{P}{Q} = \frac{[E\,I]}{\mathrm{E\,I}} = \frac{[E\,I]^{I} + j\,[E\,I]^{j}}{\mathrm{E\,I}}$$

$$= \frac{\sum_{1}^{\infty}{}_{2n-1}\,(e_n^{I}\,i_n^{I} + e_n^{II}\,i_n^{II}) + \sum_{1}^{\infty}{}_{2n-1}\,j_n\,(e_n^{II}\,i_n^{I} - e_n^{I}\,i_n^{II})}{\sqrt{\sum_{1}^{\infty}{}_{2n-1}\,(e_n^{I\,2} + e_n^{II\,2})\,\sum_{1}^{\infty}{}_{2n-1}\,(i_n^{I\,2} + i_n^{II\,2})}}$$

$$= p + \sum_{1}^{\infty}{}_{2n-1}\,j_n\,q_n$$

takes in the circuit of the general alternating wave the same position as power factor and inductance factor with the sine wave.

$U = \frac{P}{Q}$ may be called the 'circuit factor."

It consists of a real term p, the power factor, and a series of imaginary terms $j_n\,q_n$, the inductance factors of the individual harmonics.

The absolute value of the circuit factor:

$$u = \sqrt{p^2 + (\sum_{1}^{\infty}{}_{2n-1}\,q_n)^2}$$

as a rule, is < 1.

Some applications of this symbolism will explain its mechanism and its usefulness more fully.

1*st Instance:* Let the E.M.F.:

$$E = \sum_{1}^{5}{}_{2n-1}\,(e_n^{I} + j_n\,e_n^{II})$$

be impressed upon a circuit of the impedance:

$$Z = r - j_n \left(n\, x_m - \frac{x_c}{n} \right)$$

that is, containing resistance r, inductive reactance x_m and capacity reactance x_c in series.

Let:

$e_1^{1} = 720$ $\qquad e_1^{11} = 540$

$e_3^{1} = 283$ $\qquad e_3^{11} = -283$

$e_5^{1} = -104$ $\qquad e_5^{11} = 138$

or:

$e_1 = 900$ $\qquad \tan \omega_1 = .75$

$e_3 = 400$ $\qquad \tan \omega_3 = -1$

$e_5 = 173$ $\qquad \tan \omega_5 = -1.33$

It is thus in symbolic expression:

$Z_1 = 10 + 80\, j_1$ $\qquad z_1 = 80.6$

$Z_3 = 10$ $\qquad z_3 = 10$

$Z_5 = 10 - 32\, j_5$ $\qquad z_5 = 33.5$

and, E.M.F.:

$$E = (720 + 540\, j_1) + (283 - 283\, j_3) + (-104 + 138\, j_5)$$

or absolute:

$$E = 1000$$

and current:

$$I = \frac{E}{Z} = \frac{720 + 540\, j_1}{10 + 80\, j_1} + \frac{283 - 283\, j_3}{10} + \frac{-104 + 138\, j_5}{10 - 32\, j_5}$$

$$= (7.76 - 8.04\, j_1) + (28.3 - 28.3\, j_3) + (-4.86 - 1.73\, j_5)$$

or, absolute:

$$I = 41.85$$

of which is of fundamental frequency: $I_1 = 11.15$

" " " " triple " $I_3 = 40$

" " " " quintuple " $I_5 = 5.17$

The total apparent power of the circuit is:

$$Q = \text{E I} = 41{,}850$$

The true power of the circuit is:

$$P^1 = [E\,I]^1 = 1240 + 16{,}000 + 270$$

$$= 17{,}510$$

the wattless power:

$$j\,P^j = j\,[E\,I]^j = 10{,}000\,j_1 - 850\,j_5$$

thus, the total power:

$$P = 17{,}510 + 10{,}000\,j_1 - 850\,j_5$$

That is, the wattless power of the first harmonic is leading that of the third harmonic zero, and that of the fifth harmonic lagging.

$$17{,}510 = \text{I}^2\,r, \text{ as obvious.}$$

The circuit factor is:

$$U = \frac{P}{Q} = \frac{[E\,I]}{\text{E I}}$$

$$= .418 + .239\,j_1 - .0203\,j_5$$

or, absolute:

$$u = \sqrt{.418^2 + .2593^2}$$

$$= .492$$

The power factor is:

$$p = .418$$

The inductance factor of the first harmonic is: $q_1 = .239$, that of the third harmonic $q_3 = 0$, and of the fifth harmonic $q_5 = -.0203$.

Considering the waves as replaced by their equivalent sine waves, from the sine wave formula:

$$p^2 + q_0^2 = 1$$

the inductance factor would be:

$$q_0 = .914$$

and the phase angle:

$$\tan \omega = \frac{q_0}{p} = \frac{.914}{.418} = 2.8, \qquad \omega = 65.4^0$$

giving apparently a very great phase displacement, while in reality, of the 41.85 amperes total current, 40 amperes (the current of the third harmonic) are in phase with their E.M.F.

We thus have here a case of a circuit with complex harmonic waves which can not be represented by their equivalent sine waves. The relative magnitudes of the different harmonics in the wave of current and of E.M.F. differ essentially, and the circuit has simultaneously a very low power factor and a very low inductance factor, that is a low power factor exists without corresponding phase displacement, the circuit factor being less than one-half.

Such circuits for instance are those including alternating arcs, reaction machines, synchronous induction motors, reactances with over-saturated magnetic circuit, high potential lines in which the maximum difference of potential exceeds the voltage at which brush discharges begin, etc. Such circuits can not correctly, and in many cases not even approximately, be treated by the theory of the equivalent sine waves, but require the symbolism of the complex harmonic wave.

2nd Instance: A condenser of capacity $C_0 = 20$ M.F. is connected into the circuit of a 60-cycle alternator giving a wave of the form:

$$e = E\,(\cos \varphi - .10 \cos 3\,\varphi - .08 \cos 5\,\varphi + .06 \cos 7\,\varphi)$$

or, in symbolic expression:

$$E = e\,(1_1 - .10_3 - .08_5 + .06_7)$$

The synchronous impedance of the alternator is:

$$Z_0 = r_0 - j_n\, n\, x_0 = .3 - 5\, n\, j_n$$

What is the apparent capacity C of the condenser (as calculated from its terminal volts and amperes), when connected directly with the alternator terminals, and when connected thereto through various amounts of resistance and inductive reactance.

The capacity reactance of the condenser is:

$$x_c = \frac{10^6}{2\,\pi\,N\,C_0} = 132 \text{ ohms.}$$

or, in symbolic expression:

$$+j_n\,\frac{x_c}{n} = \frac{132}{n}\,j_n$$

Let:

$Z_1 = r - j_n\,n\,x$ = impedance inserted in series with the condenser.

The total impedance of the circuit is then:

$$Z = Z_0 + Z_1 + j_n\,\frac{x_c}{n}$$

$$= (.3 + r) - j_n\left([5 + x]\,n - \frac{132}{n}\right)$$

The current in the circuit is:

$$I = \frac{E}{Z}$$

$$= e\left[\frac{1}{(.3+r)-j(x-132)} - \frac{.1}{(.3+r)-j_3(3x-29)}\right.$$

$$\left. - \frac{.08}{(.3+r)-j_5(5x-1.4)} + \frac{.06}{(.3+r)-j_7(7x+16.1)}\right]$$

and the E.M.F. at the condenser terminals:

$$E_1 = j_n\,\frac{x_c\,I}{n}$$

$$= e\left[\frac{132\,j_1}{(.3+r)-j_1(x-132)} - \frac{4.4\,j_3}{(.3+r)-j_3(3x-29)}\right.$$

$$\left. - \frac{2.11\,j_5}{(.3+r)-j_5(5x-1.4)} + \frac{1.13\,j_7}{(.3+r)-j_7(7x+16.1)}\right]$$

thus the apparent capacity reactance of the condensers:

$$x_1 = \frac{\mathrm{E}_1}{\mathrm{I}}$$

and the apparent capacity:

$$C = \frac{10^6}{2\,\pi\,N\,x_1}$$

(*a.*) $x = 0$: Resistance r in series with the condenser. Reduced to absolute values, it is:

$$\frac{1}{x_1^2} = \frac{\dfrac{1}{(.3+r)^2+17424} + \dfrac{.01}{(.3+r)^2+841} + \dfrac{.0064}{(.3+r)^2+1.96} + \dfrac{.0036}{(.3+r)^2+259}}{\dfrac{17424}{(.3+r)^2+17424} + \dfrac{19.4}{(.3+r)^2+841} + \dfrac{4.45}{(.3+r)^2+1.96} + \dfrac{1.28}{(.3+r)^2+259}}$$

(*b.*) $r = 0$: Inductive reactance x in series with the condenser. Reduced to absolute values, it is:

$$\frac{1}{x_1^2} = \frac{\dfrac{1}{.09+(x-132)^2} + \dfrac{.01}{.09+(3x-29)^2} + \dfrac{.0064}{.09+(5x-1.4)^2} + \dfrac{.0036}{.09+(7x+16.1)^2}}{\dfrac{17424}{.09+(x-132)^2} + \dfrac{19.4}{.09+(3x-29)^2} + \dfrac{4.45}{.09+(5x-1.4)^2} + \dfrac{1.28}{.09+(7x+16.1)^2}}$$

From $\frac{1}{x_1^2}$ are derived the values of apparent capacity:

$$C = \frac{10^6}{2\,\pi\,N\,x_1}$$

and plotted in Fig. 4 for values of r and x respectively varying from 0 to 22 ohms.

As seen, with neither additional resistance nor reactance in series to the condenser, the apparent capacity with this generator wave is 84 M.F., or 4.2 times the true capacity, and gradually decreases with increasing series resistance, to $C = 27.5$ M.F. $=$ 1.375 times the true capacity at $r = 13.2$ ohms or $\frac{1}{10}$ the true capacity reactance, with $r = 132$ ohms or with an additional resistance equal to the capacity reactance, $C = 20.5$ M. F. or only 2.5% in excess of the true capacity C_0, and at $r = \infty$, $C = 20.3$ M. F. or 1.5% in excess of the true capacity.

With reactance x but no additional resistance r in series, the apparent capacity C rises from 4.2 times the true capacity at $x = 0$, to a maximum of 5.03 times the true capacity or $C =$

100.6 M. F. at $x = .28$, the condition of resonance of the fifth harmonic, then decreases to a minimum of 27 M. F. or 35% in excess of the true capacity, rises again to 60.2 M. F. or 3.01 times the true capacity at $x = 9.67$, the condition of resonance with the third harmonic, and finally decreases, reaching 20 M. F. or the true capacity at $x = 132$ or an inductive reactance equal to the capacity reactance, then increases again to 20.3 M.F. at $x = \infty$.

This rise and fall of the apparent capacity is within certain limits independent of the magnitude of the higher harmonics of the generator wave of E. M. F., but merely depends upon their presence, and it thus follows that the true capacity of a condenser cannot even approximately be determined by measuring volts and amperes if there are any higher harmonics present in the generator wave, except by inserting a very large resistance or reactance in series to the condenser.

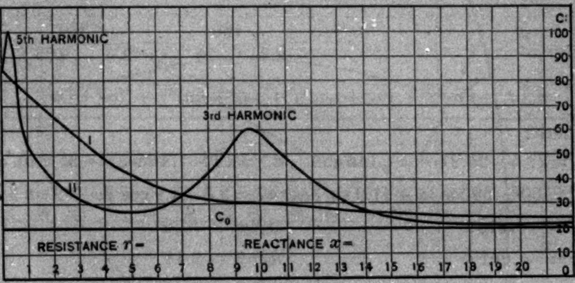

FIG. 4. Capacity $C_0 = 20$ *mf* in circuit of Generator
$\mathfrak{E} = E\,(1 - .1 - .08 + .06)$ of impedance
$Z_0 = .3 - 5\,j_n\,n$ with resistance r (*I*)
or reactance x (*II*) in series.

3rd Instance: An alternating current generator of the wave:

$$E_0 = 2000\,[1_1 + .12_3 - .23_5 - .13_7]$$

and of synchronous impedance:

$$Z_0 = .3 - 5\,n\,j_n$$

feeds over a line of impedance:

$$Z_1 = 2 - 4\,n\,j_n$$

a synchronous motor of the wave:

$$E_1 = 2250 \left[(\cos \omega + j_1 \sin \omega) + .24 \, (\cos 3 \, \omega + j_3 \sin 3 \, \omega) \right]$$

and of synchronous impedance:

$$Z_2 = .3 - 6 \, n \, j_n$$

The total impedance of the system is then:

$$\begin{aligned} Z &= Z_0 + Z_1 + Z_2 \\ &= 2.6 - 15 \, n \, j_n \end{aligned}$$

thus the current:

$$I = \frac{E_0 - E}{Z}$$

$$= \frac{2000 - 2250 \cos \omega - 2250 j_1 \sin \omega}{2.6 - 15 \, j_1} + \frac{240 - 540 \cos 3\omega - 540 j_3 \sin 3\omega}{2.6 - 45 \, j_3}$$

$$- \frac{460}{2.6 - 75 \, j_5} - \frac{260}{2.6 - 105 \, j_7}$$

$$= (a_1^1 + j_1 a_1^{11}) + (a_3^1 + j_3 a_3^{11}) + (a_5^1 + j_5 a_5^{11}) + (a_7^1 + j_7 a_7^{11})$$

where:

$a_1^1 = 22.5 - 25.2 \cos \omega + 146 \sin \omega$ $a_1^{11} = 130 - 146 \cos \omega - 25.2 \sin \omega$

$a_3^1 = .306 - .69 \cos 3\omega + 11.9 \sin 3\omega$ $a_3^{11} = 5.3 - 11.9 \cos 3\omega - .69 \sin 3\omega$

$a_5^1 = - .213$ $a_5^{11} = - 6.12$

$a_7^1 = - .061$ $a_7^{11} = - 2.48$

or, absolute:

1st. harmonic:

$$a_1 = \sqrt{{a_1^1}^2 + {a_1^{11}}^2}$$

3rd. harmonic:

$$a_3 = \sqrt{{a_3^1}^2 + {a_3^{11}}^2}$$

5th. harmonic:

$$a_5 = 6.12$$

7th. harmonic:

$$a_7 = 2.48$$

$$I = \sqrt{a_1^2 + a_3^2 + a_5^2 + a_7^2}$$

while the total current of higher harmonics is:

$$I_0 = \sqrt{a_3^2 + a_5^2 + a_7^2}$$

The true input of the synchronous motor is:

$$P_1 = [E_1 I]^1$$

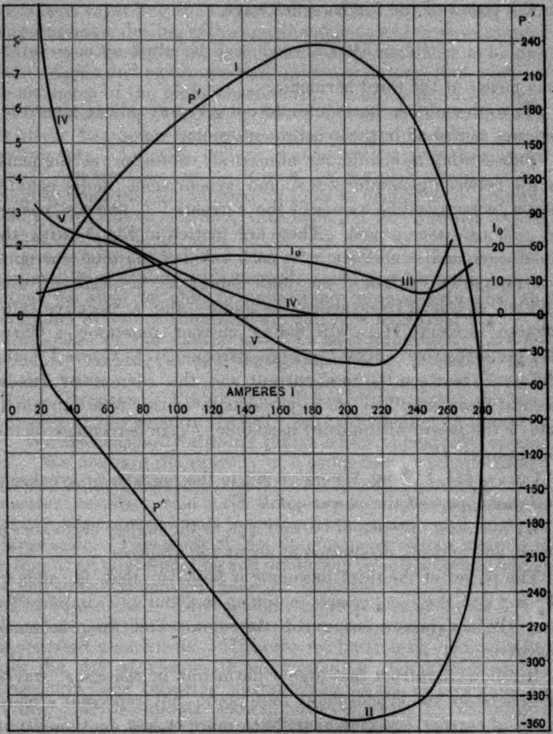

Fig. 5. Synchronous Motor.

$$\mathfrak{E}_1 = (\cos.\ \omega + j_1 \sin.\ \omega) + .24\ (\cos.\ 3\ \omega + j_3 \sin 3\ \omega.)$$

Operated from Generator.

$$\mathfrak{E}_0 = 2000\ (1 + .12 - .23 - .13.)$$

Over total impedance.

$$Z_n = 2.6 - 15\ i_n\ n.$$

$$= (2250\, a_1^{1} \cos \omega + 2250 a_1^{11} \sin \omega) + (540 a_1^{1} \cos 3\omega + 540 a_3^{11} \sin 3\omega)$$

$$= P_1^{1} + P_3^{1}$$

$$P_1^{1} = 2250\, (a_1^{1} \cos \omega + a_1^{11} \sin \omega)$$

is the power of the fundamental wave,

$$P_3^{1} = 540\, (a_3^{1} \cos 3\, \omega + a_3^{11} \sin 3\, \omega)$$

the power of the third harmonic.

The 5th and 7th harmonics do not give any power, since they are not contained in the synchronous motor wave.

Substituting now different numerical values for ω the phase angle between generator E.M.F. and synchronous motor counter E.M.F., corresponding values of the currents I I_0, and the powers P_1, P_1^1, P_3^1 are derived. These are plotted in Fig. 5 with the total current I as abcissae. To each value of the total current I correspond two values of the total power P^1, a positive value plotted as Curve I —synchronous motor—and a negative value plotted as Curve II — alternating current generator—. Curve III gives the total current of higher frequency I_0, Curve IV, the difference between the total current and the current of fundamental frequency, I — a_1, in percentage of the total current I, and *V* the power of the third harmonic, P_3^1, in percentage of the total power P^1.

Curves III, IV and V correspond to the positive or synchronous motor part of the power curve P^1.

As seen, the increase of current due to the higher harmonics is small, and entirely disappears at about 180 amperes.

The power of the third harmonic is positive, that is, adds to the work of the synchronous motor up to about 140 amperes or near the maximum output of the motor and then becomes negative.

It follows herefrom that higher harmonics in the E.M.F. waves of generators and synchronous motors do not represent a mere waste of current, but may contribute more or less to the output of the motor. Thus at 75 amperes total current, the percentage of increase of power due to the higher harmonic is equal to the increase of current, or in other words the higher harmonics of current do work with the same efficiency as the fundamental wave.

DISCUSSION.

THE PRESIDENT:—The paper is now before the INSTITUTE for discussion.

DR. LOUIS BELL:—It strikes me that the most important proposition which Mr. Steinmetz has made is in calling attention to the facts regarding the imperfection of the ordinary sine wave computations in dealing with some phenomena which we encounter in those cases having combined capacity and inductance as in long lines. Again and again we find evidence in long line work of the existence of the higher harmonics. I think more and more in a practical way it is being forced on our attention that the variations from the sine waves make themselves felt in a very striking way. I firmly believe that by recognizing these variations in the treatment of the subject by complex quantities and bringing them into the summation so as to deal at least approximately with the conditions, thus bringing the analysis to bear on that one point, we shall be able to clear up a great many things now unexplained, and also very possibly to lick into shape the practice of dealing with currents containing harmonic factors so that we shall be able to understand more clearly what the conditions of minor resonance are and how to correct them. It is this phase of the matter which is most important. We are constantly running in practice against these variations from the sine wave.

PROF. THOMSON:—I think Dr. Bell has hit the nail on the head in regard to this paper. He has voiced my opinion in regard to it. We are constantly striking things which appear to us anomalies. We measure the capacity of a condenser by an alternating current, we find that the result is all wrong, we can't get any consistent results. And such things we are meeting constantly, and Mr. Steinmetz himself is constantly meeting them I know on a large scale. This work of Mr. Steinmetz gives us a method of getting at an understanding and predicting results where formerly we had to experiment and take our chances.

THE PRESIDENT:—The excellent paper before us marks an important step in the development of the vector analysis of alternating-current phenomena. Hitherto we have been compelled to limit the use of vectors to electromotive and magnetomotive forces, and their offspring currents and fluxes, and exclude activities. Mr. Steinmetz shows, however, that by adopting a separate vector plane for power we may also represent it by a vector having real and imaginary components corresponding to the real and wattless components. The advantages of this means of representing power are considerable, and we have only to take care not to confuse the plane of single frequency in which we draw our forces and currents, with the plane of double-frequency in which we draw our powers. The two planes must be kept distinct in the mind and the algebraic laws they follow are also distinct.

Prof. W. E. Goldsborough:—I should like to say a word as coming from our universities to express how much we are indebted to Mr. Steinmetz for giving us a method, differing from the older methods we have used, which enables us to place before our students in a much more simple form many of the problems in electrical engineering. It is rather startling to the student at first to find that complex quantities admit of so direct and simple an application, but he is always charmed with the result. I doubt if Mr. Steinmetz fully appreciates the effect of his work in modifying the curricula of our American schools of electrical engineering; and in connection with this paper as in connection with the other valuable papers which he has given us I am glad to express the sentiment of our gratitude to him.

Prof. C. A. Adams, Jr.:—I wish to add one more word of gratitude for Mr. Steinmetz's valuable addition to his very valuable work along this line. I wish also to emphasize what Dr. Bell has said in regard to the practical importance of this subject of harmonics. Problems are continually arising in practice where they play a very active part, and this is not only true in long distance transmission lines, but also in the underground alternating current distribution in our large cities, particularly in Boston where the amount of such distribution is very considerable. Even in the case of the comparatively small fluctuations of the arc lighting current, there are strong indications of trouble from the same source.

Mr. T. J. Johnston:—I am not competent to criticize Mr. Steinmetz's paper, for like the famous Scotch joke, I have followed it with difficulty, but it seems to me that there is an error in the induction motor curves; taking Figs. 1, 2 and 3, the percentage of synchronism is given, for example, as one-tenth of one per cent. whereas it should be ten per cent.; in other words all of these figures are given one-one-hundredth of what they should be; I think Mr. Steinmetz will corroborate that.

Some time ago Mr. Steinmetz explained to me in a general way the fact that the general alternating wave contained only odd harmonics, that is, as generated by dynamo and motor. I do not know whether that is so in relation to all alternating current apparatus, for example, the transformal and induction regulator; that is, I do not know whether there are even frequencies introduced into the wave by apparatus other than rotating armatures. I would like to have Mr. Steinmetz say whether that is true or not.

The President:—I would like to call attention to a point of terminology which may be important in the future and which ought to be cleared up. There is no doubt about the point in the paper, because the meaning is clearly defined. The term "torque" is used with a meaning which seems misleading, at least to those who may not have the definition before them. It is stated to be the product of the E. M. F. into the quadrature component of secondary current. But this product is not a "torque" or "moment" but is a "power" or "activity," and it is defined as the power which the torque develops at synchronous speed. Now a torque cannot be both a torque and a power, because the one is a moment of a force about an axis and the other is a rate of working. Further on in the paper the "torque proper" is defined as this so-called synchronous torque divided by an angular velocity and by the number of poles. Surely it would be better to retain the term "torque" for this its usual meaning, and use some such term as "synchronous power" for the activity of the torque at synchronism.

Mr. Steinmetz:—Taking up the last question first, I appreciate that it is undesirable to denote by the same name two quantities of different dimensions. I was obliged, however, to introduce the definition of torque in synchronous watts as the only one suitable for general theoretical discussions, by its independence of individual conditions of the apparatus, as number of poles, frequency, etc.

In synchronous watts, torque has as definite a meaning as power in watts. To avoid the objection of the different dimension from torque as usually given in foot pounds, the value $T = e^2 a^1$, etc., may be divided by the frequency: $T_0 = \frac{e^2 a^1}{N}$ as indeed is implied by "synchronous" watts, or a new name may be invented for the quantity $T = e^2 a^1$ if anybody can suggest a suitable name.

The lettering on Figs. 1 and 2 is obviously a misprint which will be corrected, it should read "in fractions" and not "in per centages."

I would like to draw your attention to one curious and somewhat unexpected feature regarding the effect of higher harmonics of E. M. F. on the current input of the condenser.

The current input, and thus the apparent capacity of a condenser, when supplied by a wave of E. M. F., containing higher harmonics, is independent of the magnitude of the higher harmonics, provided that the resistance between generator and condenser is negligible, and depends upon the existence of these harmonics only. Thus a quintuple harmonic of 25% of the fundamental will send neither more nor less current into a condenser at the same effective terminal voltage than a quintuple harmonic of 5% or less, at negligible resistance.

The only effect of the magnitude of higher harmonics is that the resistance which is negligible with a higher harmonic of 25% of the fundamental, may not be negligible with a higher harmonic of 5%, or in other words the lower the amplitude of the higher harmonics the lesser resistance is sufficient to suppress their effect on the condenser.

Dr. Bell:—In this matter of torque, I must beg of Mr. Steinmetz out of kindness to the profession not to invent a new name. We have now not a deficit but a plethora of new names and I think that a little clearer explanation would avoid what I should style a real calamity.

In respect to this matter of harmonics, one quickly realizes that when it comes to a matter of resonance, when there is true resonance mechanical or of any other sort the original amplitude of the wave of energy does not make very much difference. It is the resistance to the propagation of the vibrations, whatever they may be, that determines the magnitude of resonance. So while it seems queer that a small fifth harmonic should raise the very deuce with the capacity of a condenser, we should remember that this condition expresses a general physical fact not at all confined to electrical problems.

Prof. H. J. Ryan:—May I say in answer to the gentleman who spoke of the even harmonics that the positive and negative magnetic flux must always have the same value. The E. M. F. values will therefore be alike and such values can therefore always be made up of components and the fundamental and the odd numbers of harmonics. The even numbers would represent a state of affairs in which the flux positive and negative would not be the same.

Prof. E. Thomson:—There is just one point about the production of the even harmonics. We could provide for them without doubt by making a single conductor travel in front of magnetic poles of different width, say the north poles of a certain width and the south a greater or less width, and a single conductor would thus operate to produce the ordinary wave containing even harmonics.

Mr. T. J. Johnston:—We may take a vibrating string in music and split the vibration so that there will be two nodes, making the string vibrate in three parts, giving the third harmonic. Take another string and split that into two parts, giving rise to the second harmonic or octave, and the vibrations may be so mingled as to sound in the same wave both the second and third harmonics.

What I want to know is, is there anything analogous to that in the alternating circuit; by which, for example, by power from the outside a second harmonic may be introduced into that circuit, so that a second and third harmonic will flow in the same wave?

Dr. Samuel Sheldon:—In instrumental music the duration of the different tones is represented by notes of different lengths as eighths, sixteenths and halves. There is also the tempo mark, and yet if we take a classic and place it in the hands of an inexperienced performer, although the directions on the printed page are followed perfectly, the resulting music is in no wise of the same character as if played by an artist. The difference between the performances of the two, the artist and the unskilled performer, is largely a question of allowance in time. There is a subdivision of the different time intervals far below the limits which are represented on the printed music. These subdivisions are readily detected by the ear. In the question of alternating currents of course any problem could be easily solved were it not for phase differences. If we knew the instantaneous values of current, E. M. F., torque, etc., in a number of small successive time intervals we could solve without the use of any complex quantities. However, the difference between the ordinary person and the artist I think is well represented here to-day and I would like to add my word in appreciation of the artist who has presented this paper to us and who seems to perceive these little time intervals by self-consciousness.

PROF. GOLDSBOROUGH:—There is one matter which, so far, I believe, has not been touched upon, which is the use of complex quantities in connection with the design of electrical machinery. If we have harmonics in our circuits they are usually developed in the station generator. We know that whenever we use an iron-clad armature we get harmonics. If we follow the type of the smooth core armature it is possible to produce what is practically a pure sine wave. Now, as every introduction of inductance into the circuit, whether it be from transformers or induction motors or even from changes in the flux densities used in the iron of the alternator, modifies the character of the wave, it also modifies the amplitudes of the harmonics which are component parts of the wave, and there is presented a series of mutually related phenomena the treatment of which is very difficult. I think, therefore, that this paper which Mr. Steinmetz has brought out, showing us how we can apply complex quantities to the analyzation of problems involving harmonics, is a contribution of great value. In other words, by being able to analyze our generators properly, in the process of design, and thereby pre-determining the exact character of their regulation, we will be better able to adapt them to the needs for which they are built, and cut off in the development of the machines the undesirable qualities which cause trouble.

[COMMUNICATED AFTER ADJOURNMENT BY DUGALD C. JACKSON.]

Mr. Steinmetz makes a very interesting expansion, in this paper of his "symbolic" method of representing alternating waves, and the paper is undoubtedly a valuable contribution to the literature of the subject. It is pleasing to find Mr. Steinmetz bringing his analysis into more rational paths, as his apparent disregard in the past, of the imaginary or wattless component of the power vector must have led many students to question the generality and truth of his methods.

Even now, Mr. Steinmetz uses in Part I of the paper, what appears to me an unnatural treatment of the power vector, and I therefore present for consideration a treatment abstracted from the notes of my lectures to advanced students of alternating currents. Polar coordinates are used in representing the vectors, instead of the rectangular coordinates used by Mr. Steinmetz. This is purely a matter of convenience, as the transformation from one set of coordinates to the other is readily effected. Letters in bold face type represent vectors and italics represent moduli or tensors.

The vector of electrical pressure is represented in polar coordinates by

$$\mathbf{E} = E(\cos\theta + i\sin\theta),$$

or, writing cis θ in the place of $(\cos\theta + i\sin\theta)$, to represent the direction coefficient or "turning factor" in the expression, gives

$$\mathbf{E} = E \operatorname{cis}\theta.$$

In the same way current is represented by

$$\mathbf{I} = I \operatorname{cis}(\theta - \varphi)$$

and impedance is

$$\mathbf{Z} = \frac{\mathbf{E}}{\mathbf{I}} = \frac{E}{I} \operatorname{cis}\varphi = Z \operatorname{cis}\varphi.$$

This quotient is drawn in accordance with the well-understood theorem that the ratio of two vectors has a tensor equal to the ratio of the tensors of dividend and divisor and an amplitude equal to the difference of the amplitudes of dividend and divisor.

Impedance is here shown to be a vector ratio or quaternion which has the characteristics of an operator. Its amplitude is wholly dependent upon **E** and **I**, it cannot be said to have a frequency of its own, and it clearly can have no effect upon the period of **E** and **I** which is determined by the period of θ, — a period which is in no manner dependent upon **Z**. On the other hand, Mr. Steinmetz is clearly in error when he asserts that, in vector equations, impedance, **Z**, "is a constant numerical factor" and for that reason does not affect the frequency when it operates on **I** to produce **E**.

The reciprocal of impedance, whatever its name may ultimately be, is of course,

$$\mathbf{Y} = \frac{1}{Z} \operatorname{cis} (-\varphi).$$

The power vector has the form

$$\mathbf{P} = P \operatorname{cis} \psi,$$

and it is equal to the product of the current and pressure vectors, or

$$\mathbf{P} = \mathbf{I}\,\mathbf{E} = I\,E \operatorname{cis} (2\,\theta - \varphi) = (I\,E \cos \varphi - i\,I\,E \sin \varphi) \operatorname{cis} 2\,\theta.$$

Cis ψ is therefore equal to cis 2 θ, and it is clearly seen that **P** has twice the frequency of **I** and **E**. Moreover, the apparent energy P, is shown to be composed of two rectangular components. One of these ($I E \cos \varphi$) is real and represents the true average power in the period, and the other ($i\,I\,E \sin \varphi$) is imaginary, disappears when taken over any complete half period, and represents the *wattless energy* (if such a contradictory expression is allowable) that swashes back and forth in the circuit but always gives as much as it takes in any complete half period.

By drawing a curve through the points found by giving various values to θ from 0 to 360° in the expression P cis 2 θ, we get the well known sinusoidal "power loops" which are of twice the frequency of the corresponding curves of pressure and current.

The true energy is zero when

$$\cos \varphi = 0,$$

that is, when $\varphi = 90°$, in which case **E** and **I** are in quadrature; and the *wattless energy* is zero when

$$\sin \varphi = 0,$$

that is, when $\varphi = 0°$ or $180°$, in which case **E** and **I** are either in phase or in opposition. Mr. Steinmetz's equations

$$\frac{e''}{e'} = -\frac{i'}{i''} \text{ and } \frac{e''}{e'} = \frac{i''}{i'}$$

are at once evident.

The algebraic sign of the angle φ indicates whether the *wattless energy* is the result of equivalent inductance or capacity, and its numerical value indicates whether the true power is delivered or absorbed by the circuit. The application of the commutative law to the vector formulas is not affected by doubling the amplitude in the equation.

All of the deductions which may be properly drawn from Mr. Steinmetz's discussion of the power vector may be more readily drawn from the forms given, which are not only in harmony with the ordinary analytic forms and appear to be more natural but contain indications of evident importance that do not appear in Mr. Steinmetz's forms.

The discussion may be simply and readily extended to torque and applied to the problems that Mr. Steinmetz introduces. As the important quantities in the solutions of the problems relate to the magnitude of torque, apparent energy, and true energy it is often convenient to transform into rectangular coordinates. This (using Mr. Steinmetz's notation) gives for apparent energy

$$P = P^1 + P^j = (e' i' + e'' i'') \pm j (e'' i' - e' i''),$$

where the plus or minus sign should be used between the two terms of the last member of the equation as φ is essentially positive or essentially negative.

The further transformations readily follow and it is unnecessary to extend the discussion for the purpose of showing them. Possibly it is needless to add that an analogous treatment is equally simple when applied to the matter of Part II of Mr. Steinmetz's paper.

THE CHAIRMAN:—We will pass to the next paper on the programme, which is "The Cost of Electricity in Some Typical Buildings in New York City," by Percival Robert Moses.

A paper presented at the Sixteenth General Meeting of the American Institute of Electrical Engineers, Boston, June 20th, 1899, President Kennelly in the Chair.

THE COST OF ELECTRICITY IN SOME TYPICAL BUILDINGS IN NEW YORK CITY.

BY PERCIVAL ROBERT MOSES.

For several years I have been questioned as to the cost of electricity in isolated plants and frequently the statement was volunteered that "of course the central station supply can offer much better terms." I have partially investigated the conditions existing in over a hundred buildings of various sizes and types, and as the subject is one of general interest to engineers and laymen. I have ventured to prepare a paper embodying the results I have found, and trust that the discussion thereof, will bring additional figures of value.

I have thought it best to confine my paper to a few typical buildings, which I know all about, and to discuss these in detail, rather than to tabulate figures more or less accurate from a large number of buildings, where all the conditions were not obtainable.

The methods by which the figures have been arrived at, are of course, of the highest importance, in fact, they entirely determine the value of the results.

In all cases except where otherwise stated, the kilowatt hours supplied by the isolated plant have been obtained, either from hourly and daily reports of ammeter readings, or from wattmeter readings.

The other items of coal, water, oil, waste, lamps, labor, etc. have been furnished me by the owners, from their books, or in one case, by the agent in charge of the building. I have laid great stress on the necessity of obtaining accurate results, and as the question was one of interest to all who run plants, I experienced little difficulty in obtaining access to the books.

As the question of distribution and character of load has a considerable influence on the fuel consumption and the labor charge, I have included load diagrams from many of the plants, obtained as stated either from logbooks and hourly records, or from personal readings.

Before proceeding with the figures and diagrams, a definition and an explanation are necessary. What is the "cost of electric lighting or electricity" in a building plant? In my opinion, the cost of electricity in a building plant, is the difference between the cost of supplying the other requirements of the building, (the electricity being purchased outside), and the cost of supplying the other requirements of the building and electricity from private plant. The cost in all cases to include interest and depreciation. Interest and depreciation are reckoned at 5% each. Money is invested in a plant and can generally be replaced at an interest charge of 5%. The plant may be considered worthless in fifteen years, as an average case, and enough must be put aside yearly to repay this amount of its cost. 5% per annum with interest at 5% will equal the original investment in 15 years.

No cost per kilowatt hour can be definitely fixed, as every case must be determined for itself. For instance; the cost of electricity where steam is obtained from a central service may be quite different from the cost where boilers are operated. The addition of the electric plant in the former case may cause a large and expensive increase in the labor and rental charges while in the latter case no increase in these items might be necessary. The cost of electricity in a building, where high-speed elevators, refrigerating and laundry machinery are operated, in addition to the electric plant, is much less per kilowatt than it is in a "loft" building, where skilled supervision was not necessary before the introduction of an electric plant. The necessity of steam for heating has an important influence on the cost, and on this account, figures for buildings in New York would be of little value in New Orleans or Toronto.

I have chosen for discussion in the paper, data from hotels, office buildings, loft buildings, department stores and apartment houses.

Considering the hotels, and with proper respect to height, the tall one first: This hotel is 15 stories in height and is an example of the finest and best in the hotel line, although the rapid improvement and development of the past two years has made even

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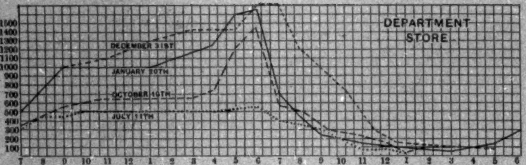
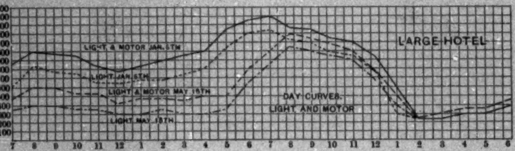


Fig. 2.



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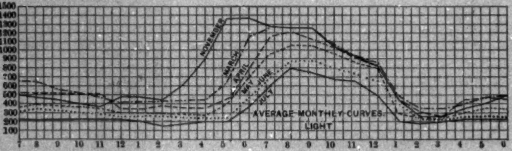

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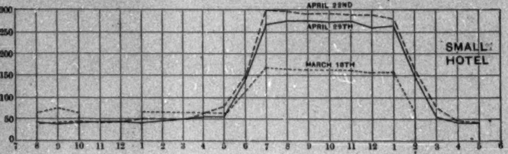

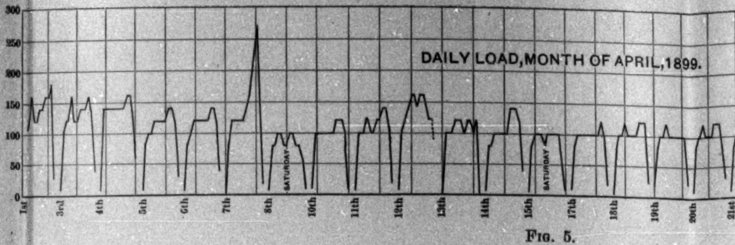

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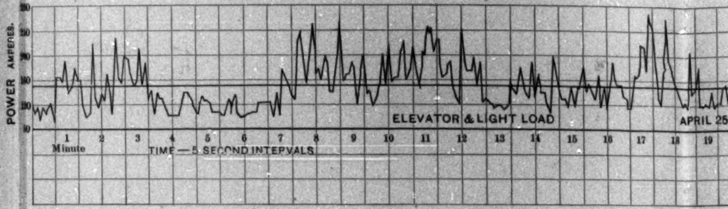

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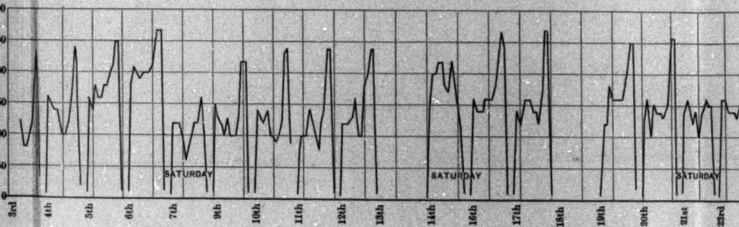

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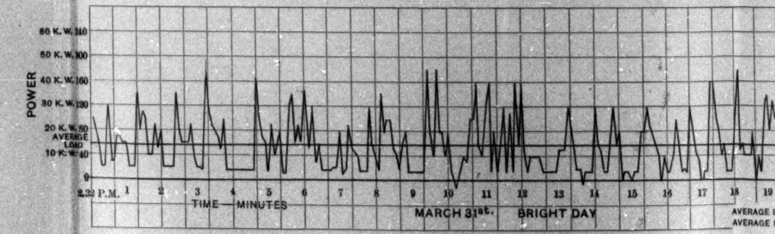

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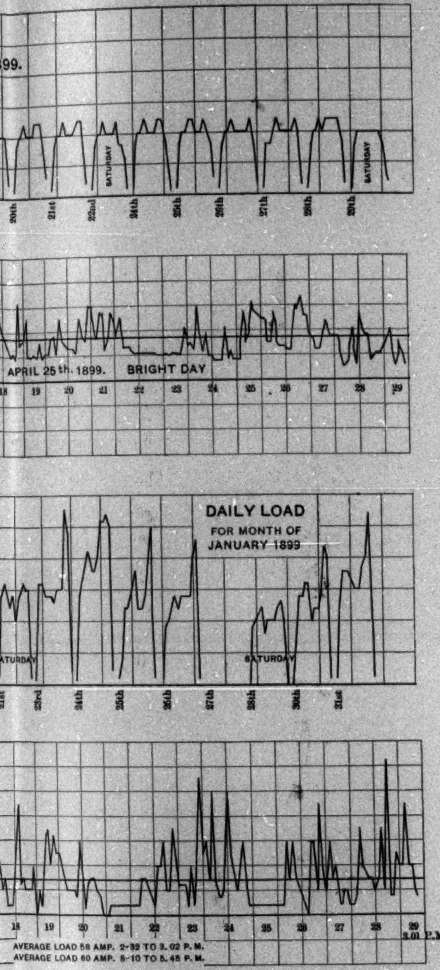
99.
SATURDAY
SATURDAY
20th
21st
22nd
24th
25th
26th
27th
28th
29th
APRIL 25th 1899. BRIGHT DAY
18
19
20
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24
25
26
27
28
29
DAILY LOAD
FOR MONTH OF
JANUARY 1899
SATURDAY
SATURDAY
23rd
24th
25th
26th
27th
28th
30th
31st
18
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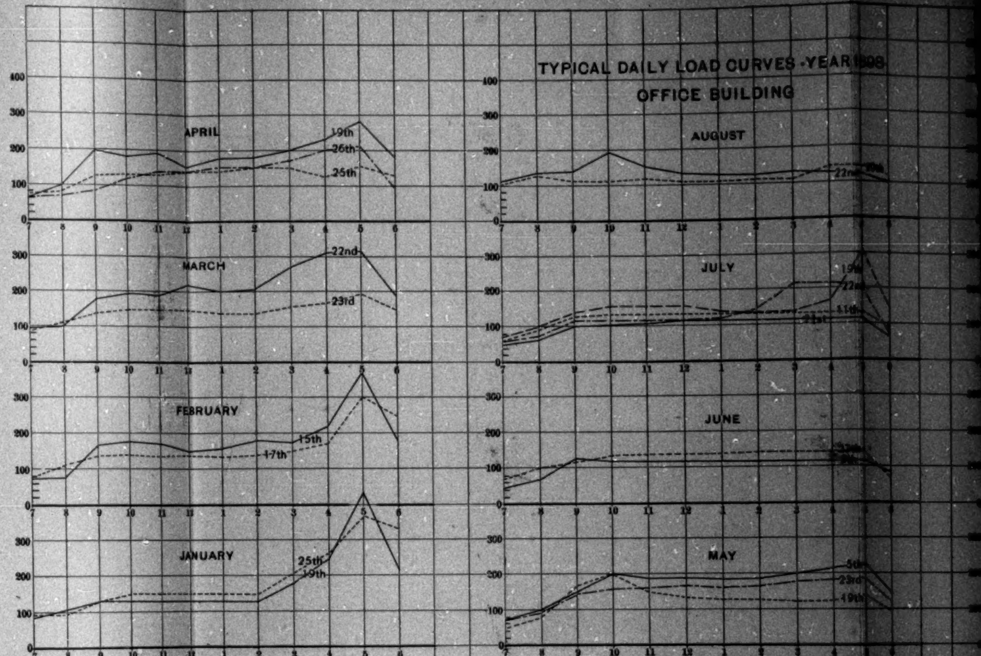
29
3.01 P.M.
AVERAGE LOAD 58 AMP. 2-32 TO 3.02 P. M.
AVERAGE LOAD 60 AMP. 5-10 TO 5.45 P. M.

Fig. 9.


TYPICAL DAILY LOAD CURVES - YEAR 1898
OFFICE BUILDING
APRIL
19th
26th
25th
MARCH
22nd
23rd
FEBRUARY
15th
17th
JANUARY
25th
19th
AUGUST
22nd
30th
JULY
19th
22nd
11th
21st
JUNE
MAY
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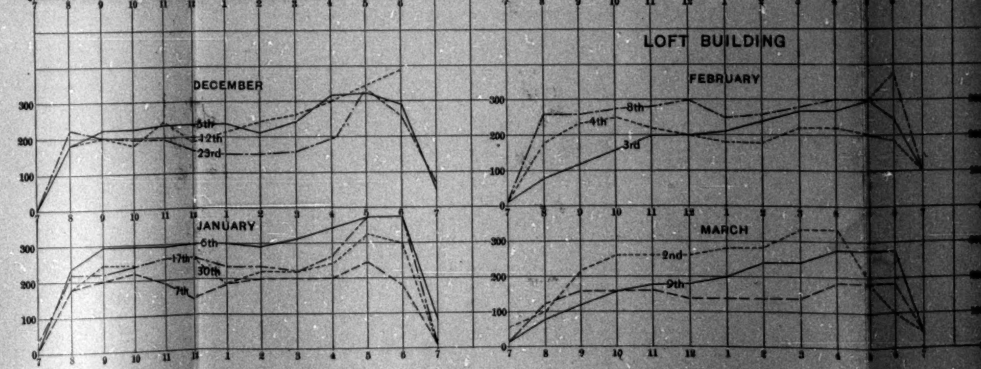
DECEMBER
5th
12th
23rd
JANUARY
5th
17th
30th
7th
LOFT BUILDING
FEBRUARY
8th
4th
3rd
MARCH
2nd
9th
100
200
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Fig. 10.

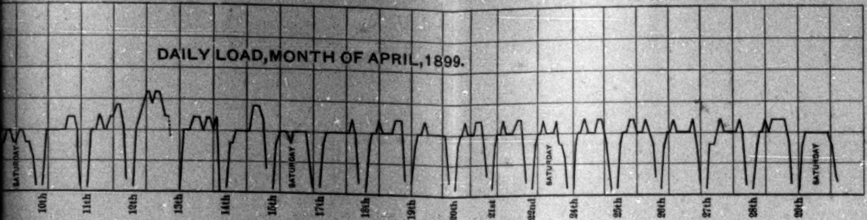


FIG. 5.

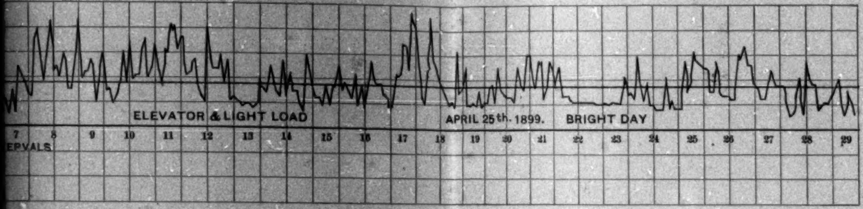
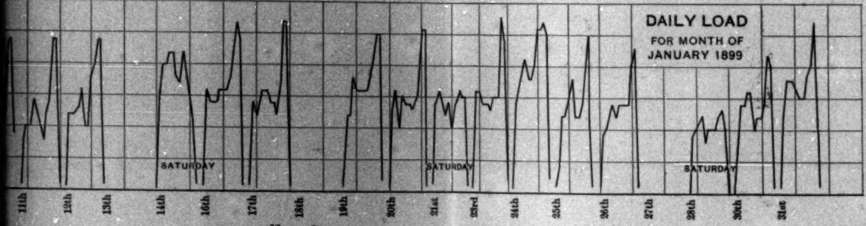


FIG. 6.



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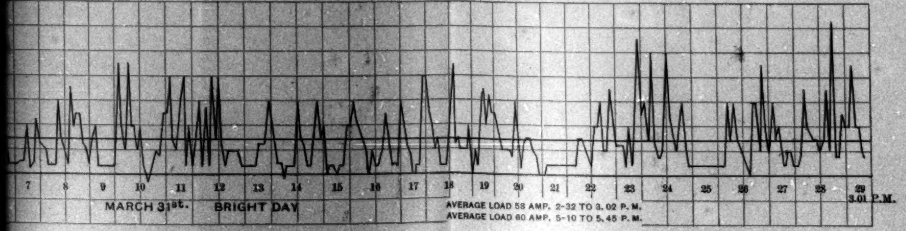


FIG. 8.

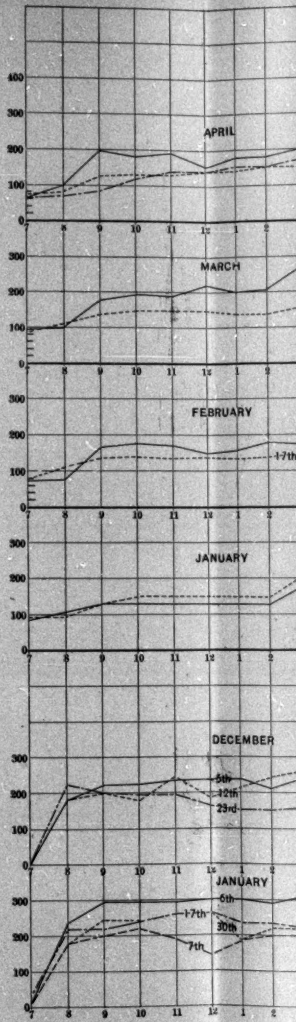
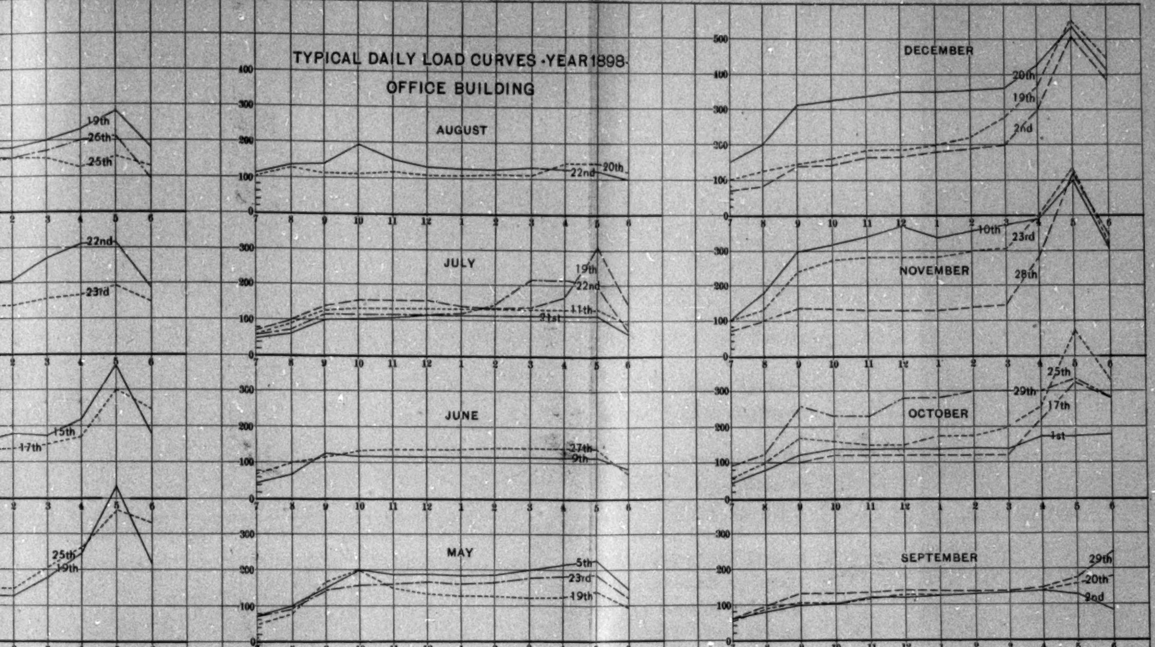


Fig. 9.



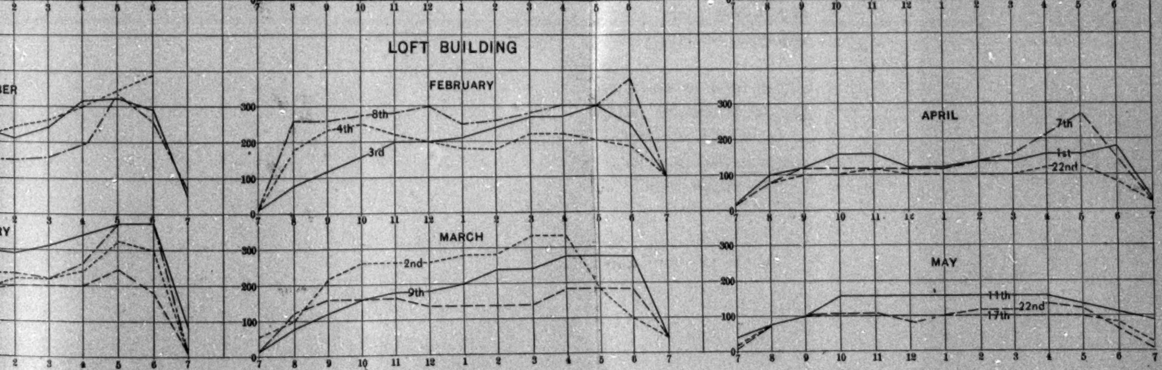


Fig. 10.

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this modern plant slightly old-fashioned. It contains 11 electric elevators, four of which are high-speed screw elevators, compound engines, directly connected to generators, water-tube boilers operating at 125 lbs. pressure and a complete refrigerating and ice-making outfit, besides all usual apparatus and laundry machinery. One of the proprietors, very courteously placed his logbooks at my disposal.

In Fig. 2 are shown curves of hourly variations in the demand for light and power. This curve does not include elevator power merely the power for motors as laundry, kitchen, office, etc. The elevator load being of the usual character and similar to that shown in Figs. 6 and 8. Curves, for which I am indebted to the engineer, of average daily lighting load are shown for six months of 1897, in Fig. 3.

The cost of the electricity in this hotel, is the difference between the cost of operating plant with the electric generating apparatus and the cost without.

A depreciation and interest charge on about one-fifth of the boiler and steam piping should be added, as about four-fifths of the boilers installed would have been necessary without the plant. This amounts to about $500 per year.

Without going into the question of what the total cost of operating this plant is, there are certain items, such as lamps, cylinder and machine oil, engine and dynamo repairs and packing which may be directly charged to electricity. These amount to $3000. In the item of labor, if an electric plant were not in operation, 1 engineer, 2 firemen and 2 oilers could be dispensed with—about $3600 per year.

The question of amount of coal used per kilowatt hour in this plant depends entirely on the season and the amount of current supplied. If the exhaust steam from the engine is insufficient to meet the heating requirements, additional lighting makes no appreciable change in the amount of coal used. During warm weather or when the steam for lighting exceeds the amount required for heating, the amount of coal used increases in proportion to the electricity supplied. The curve of coal per kilowatt hour would therefore be similar to the curve of steam generation, *i. e.*, it would be almost straight, until the heating requirements were met, and would then rapidly increase for additional kilowatt hours supplied. This variation is illustrated by the following table. [See next page.]

From April 25th to May 2nd, the hotel lighting was supplied by the Edison company, the elevators and motors being supplied from the isolated plant. From May 4th to May 11th, the whole installation was supplied from the isolated plant. As the temperatures for the two weeks were nearly the same, the work, other than the lighting, may be considered about constant, and the variation in the average amount of coal used charged to the electricity supplied.

In March and January coal per additional kilowatt hour may be estimated in a similar manner, although as the Edison service was not used, the differences are not so great and the deductions are therefore liable to large error.

Interval.	Ampere hours.	Kilowatt hours total.	Coal lbs.	Coal per additional K W. hour	Average load kilowatt.	Average Temp. at 6 P.M.
May 4–11	5439 x 24	15200	221000		83	57° F
April 25–May 2	2313 x 24	6400	195000		28	61½° F
Difference	3126 x 24	8800	26000	2.95		
March (2–6) (14–18)	7140 x 24	20250	339000		105	43° F
March (6–14)	6569 x 24	18600	329000		97	44½° F
Difference	571 x 24	1650	12000	7 25		
Jan. (15–22) (29–Feb. 5)	12719 x 24	36000	608000		107	32½° F
Jan. (1–8) (22–29)	11899 x 24	33660	593000		101	31½° F
Difference	820 x 24	2340	15000	6 4		

This table shows the kilowatt hours supplied during 2 sets of 7 days each in May, 2 sets of 8 days each in March and 2 sets of 14 days each in January. As the work done other than the electrical is about constant during any pair of sets of days the difference in amount of coal used is chargeable to the electricity supplied. Inspection of the table shows that throwing the lighting load on the Edison service from April 25 to May 2nd, reduced the coal used by less than 3 lbs. per K. W. hour. As the actual coal per K. W. hour in the non-heating season is between 3½ and 4 lbs. it is evident that during the period, April 25th–May 2nd, the exhaust steam was insufficient to heat the building and also that the additional 8800 K. W. hours supplied, after the Edison supply was cut off from May 4th to May 11th, were not a direct charge on the coal pile. The results in March and

January are as stated, subject to large errors but they serve as an indication of the influence of heating. It is evident from the table that in January and March the exhaust was in excess of the steam required for heating, while the contrary was the case from April 25th to May 2nd, when the Edison current was in use. The coal chargeable to electricity is derived as follows:

The current supplied from April 25th to May 2nd (excluding Sunday) was approximately equal to that supplied from May 4th to May 7th and also from May 8th to May 11th. By combining these figures and eliminating the electric factor, a rough idea of the constant requirement is obtained and proves to be about 24,000 pounds of coal per day, exclusive of electricity at this mild season.

Equal amounts of electricity were supplied during two periods of 13 days and of 12 days in July.

The difference in the amount of coal burned was 22,000 pounds. As the total amount of electricity supplied was the same, this difference is evidently the constant quantity of coal used, exclusive of that required for electricity. The constant for July is therefore about 22,000 pounds. This method is subject to considerable error, but by comparison with other of the non-heating months the figure obtained has been shown to be about correct.

In the same way the constant for March is fixed at 35,000 pounds coal per day. That for January is about the same.

The coal per kilowatt hour in each of the above months is the difference between the total coal used and the constant coal requirement, divided by the number of kilowatt hours.

The total coal used in 25 days in July was 700,000 pounds. The constant demand 550,000, leaving 150,000 pounds for the electricity, and as 41,500 kilowatt hours were supplied this gives a figure of 3.62 pounds per kilowatt hour.

In January, in 14 days, 608,000 pounds of coal were used, and subtracting the constant coal, *i. e.*, 14 × 35,000 pounds equals 490,000 pounds, leaves 118,000 pounds for 36,000 kilowatt hours or 3.27 pounds per kilowatt hour. The average for the year will not exceed 3.4 pounds per kilowatt hour.

The total kilowatt hours per year exceeds 825.000. The cost of electricity in this hotel is therefore,

Coal		$4,075
Labor		3,600
Sundries, (lamps, repairs, supplies, etc.)		3,000
		$10,675
Interest and depreciation		3,000
		$13,675
Cost per K. W. hour exclusive interest and depreciation	1.3	cents.
" " " " inclusive " " "	1.66	
Coal per K. W. hour	3.4	pounds.
Cost of coal per K. W. hour	0.5	cents.

By the introduction of an automatic stoker and a storage battery and the use of cheap coal with proper provision for ensuring good combustion, this figure could be further reduced, but it is pretty nearly low water mark for electricity, as supplied from an isolated plant.

A smaller hotel of which the data follows has had a plant in operation only a few months, but as a log of coal, amperes and other plant items is carefully kept hourly, its discussion will be of value. This hotel is 75x100 feet with an L 25x100 feet, seven to eight stories.

The electric plant is too small for the work, but it was installed with two understandings: 1st, that the Edison current was to be on hand for emergencies; 2nd, that no signs were to be installed. Four signs have been installed and the Edison current removed. I merely insert these facts as an example of the difficulty in meeting conditions. The plant consists of

Two H. T. 4′ × 12′ 40 H. P. boilers.
One 12′ × 7′ × 10′ Worthington elevator pump.
One 4½′ × 2¾′ × 4′ Worthington return pump.
One house pump.
One 35 K. W. engine generator combination.
One 3-ton absorption Steele & Condict refrigerating machine.

Steam is supplied to kitchen, elevators and other pumps, to engine and ice machine.

The previous cost of heating and elevator operation was

Coal, 2 tons per day, $4	$2,900
Labor, 3 men	1,320
Sundry supplies	150
Water (for purpose of comparison)	constant
	$4,370

In order to determine the cost of electricity the above cost should be reduced by $770 on account of saving obtained by use of small coal.

On this basis the cost of heat and elevator operation was $3,600. The cost of installing generating and refrigerating plant was $5,000—a plant to meet the present conditions and maintain a proper reserve would have cost $7,500, on which an interest and depreciation charge of $750 is ample.

The present cost of operation is

Coal, 3½ tons per day, at $2.75	$3,500
Labor, 4 men	2,000
Oil, waste, lamps (estimate)	300
Ammonia	50
Water, constant	500
	$6,350
Interest and depreciation	750
Total cost	$7,100
Previous cost	3,600
Cost of electricity, refrigeration and steam for kitchen	$3,500

The light used averages 2,500 ampere hours per day at 118 volts equal to 2,150,000 — 16 C. P. equivalents in a year.

The refrigeration varies from 1½ to 2 tons per day, ice melted equivalents = 640 tons per year.

The cost of these two items at $\frac{3}{10}$ cents per 16 C. P. equivalent and $3 per ton for ice would be $8,370 per year.

This shows a net saving of about $5,000 per year by the isolated plant. To obtain a figure for the cost of electricity in this plant, the cost of water used for condensing the ammonia gas into liquid ammonia, the interest and depreciation on the cost of refrigerating apparatus and the cost of ammonia must be deducted. These amount to $650.

By a test made on the refrigerating machine, I found that 50 pounds of steam per hour per ton actual refrigeration was sufficient to allow, and on this basis the coal used for refrigerating amounts to about 100 tons or $275 in the year—the hours of operation being averaged from the log records. The labor increase may be divided between the electric and refrigerating apparatus in proportion to the coal used *i. e.*, four-fifths and one-fifth.

The cost of electricity supplied is therefore:

Coal, 450 tons at $2.75	$1240
Labor, $660 x ⅘	530
Oil, waste and lamps	275
Water	50
Interest and depreciation	550
	$2645

Cost per K. W. hour is $\frac{2645 \times 20}{2,150,000}$	2.45 cents
Coal per K. W. hour $\frac{900000 \times 20}{2,150,000}$	8.37 ponnds
Cost of coal per K. W. hour	1.17 cents

The load curves shown in Fig. 4 are characteristic of this type of hotel, or more properly of the restaurant, and in general outline will be duplicated in any restaurant having illuminated signs and decorative lighting. Its characteristics are, a steady light load during twelve hours and a steady heavy load during ten hours—conditions best met by two units, or in some cases three.

As an example of what an engine can do when necessary, it is worth noting that this plant has run on an average over 20 hours a day for the past four months, with an eight-hour daily overload of from twelve to twenty per cent. The use of a force pump system of oiling with filtering device etc. on the engine, has amply proven its value by reducing the consumption of machine oil to less than one quart per day, about 0.00025 cent per K. W. hour.

From the hotel to many apartment houses the change is more nominal than real, as frequently in those of the more modern type electric light, refrigeration and steam heat are distributed throughout "free" to the tenants and a large restaurant on the ground floor makes the resemblance complete. There are, however, apartments having a load quite distinct from the hotel. In these, electric light is only supplied from 5 o'clock until 12 or 1 and during the rest of the time tenants burn gas, which they pay for directly. As most of the cooking is done by gas ranges, the small additional charge for light used is not objectionable. Elevator service is necessary at all times, or at least until late at night. Formerly these elevators were steam-hydraulic, but latterly the electric type has predominated. Where light is furnished free, an isolated plant is installed (unless the building is in speculative hands) but when light is not free, a plant is seldom installed. In none of these plants have I been able to obtain a complete hourly record of lights burned, but from several of the same size I have obtained careful estimates, which, as they agree very closely may be considered approximately correct. The coal, labor and similar items are exact and are taken from records. I will instance two, which are typical of their kind.

The first is a seven story apartment house, 100 x 100 feet, with 38 apartments, a small restaurant, having two hydraulic passenger elevators and one hydraulic freight elevator and a belted electric light plant. The latter operates only at night and until one o'clock. One engineer and a fireman during the day, and one man at night constitute the entire staff, and as the repairs have been nothing for the past five years, and as the service has been thoroughly satisfactory, the staff is evidently sufficient.

The total cost of operating the steam and electric plant is:

Coal	$1642
Water (estimated) 25c. per ton of coal	90
Oil, waste, etc	72
Engineer, fireman and assistant	2390
Lamps, ashes and repairs	115
Total cost	$4309

A similar apartment house having one hydraulic elevator and steam heating and outside electric service, costs:

Coal (for heating and hot water)	$1157
Labor (2 men $12 and 10 per week)	1144
Sundries (estimated)	100
	$2401

The cost of elevator and light service is therefore about $2,000 per year. The total installation is 900 incandescent lights and the average load from five until twelve, is 500 lights reduced in summer to about 350. The total number of kilowatt hours supplied by this plant is over 55,000. The cost per kilowatt hour is therefore, 3.8 cents, exclusive of interest and depreciation, which add 0.9 cents or a total of 4.7 cents per kilowatt hour. The coal per kilowatt hour is about 7 pounds. The cost at present wholesale rates would be over double the above figures, *i. e.*, about 9½ cents per K. W. hour from the central station.

In another apartment house of the same size, a low pressure heating boiler is used with electric elevators. The cost in this plant, exclusive of light and elevator power was:

Coal	$730
Labor	924
Sundries (estimated)	46
	$1700
Electric Elevators (power)	350
	$2050

In an apartment of the same size:

Coal	$2190
Labor	2100
Sundries (estimated)	300
	$4590

The cost of heating and electricity for elevator and lighting from the isolated plant was $4590.

Deducting $1,700, found as the cost in the preceding instance of heating and supervision, from $4590 leaves $2900 as the cost of elevator power and lighting. As the power for an electric elevator from the central service would not cost over $250 per year, a K. W. hour of light from this style of plant is evidently more expensive than from one with hydraulic elevators and an electric light plant, and unless some means of equalizing the load is employed, this is undoubtedly the case.

I am watching with interest a storage battery plant installed last month, and hope that therein lies the salvation of the electric elevator in apartment houses, although another means, *i. e.* electric heating, offers many advantages. In apartments smaller than those mentioned, isolated plants are installed in connection with hydraulic elevators, but low pressure heating and outside service with electric elevators is cheaper and better when it is available.

A number of experiments are being made with gas engines in apartment houses and I hope some of those present will give us some definite data regarding them. From figures I have obtained, however, the results are not very promising.

The conditions in department stores resemble those in hotels and hotel apartments. As in the hotel, there is a 24-hour demand for light with a peak from 4 until 7 o'clock, but in the store the peak drops abruptly at this time, while in the hotel the load either remains stationary or continues to increase for many hours.

Characteristic curves of a typical plant are found in Fig. 1, the data having been obtained from R. H. Macy and Company, through the courtesy of Mr. Straus and his engineer, Mr. Brennan.

As in the hotel, these stores use large quantities of steam for other than electrical purposes—in Macy's there are 17 elevators (steam and hydraulic), two restaurants, a laundry, a cash blower engine of 55 I. H. P. and a small engine for glass cutting. This varied use renders it difficult to obtain the cost of electricity, but I am able to present figures from a similar building where electricity for lighting was formerly bought from the central station and where the other conditions have not varied materially since the installation of the plant.

The load in this building, although there are fewer lights and smaller load maxima than in Macy's, about equals the total in the latter on account of several display signs which hold up the average until 10 nightly.

The additional coal burned since the installation of the plant last year is less than 925 tons (pea).

The total kilowatt hours supplied from the plant was over 200,000 and the cost of installation, including additional boilers, was not more than $20,000.

The cost of electricity was

Extra labor, 1 night engineer, 1 fireman, 1 helper and 1 electrician (6 months)	$1,200
Oil, packing, waste and lamps	450
Coal, 925 tons, at $3	2,775
Extra water (about)	250
Repairs	
	$4,675
Interest and depreciation (6 months)	1,000
Total cost	$5,675
Cost per kilowatt hour less than	2.85 cents.
Coal per kilowatt hour	9.25 pounds.
Cost of coal per kilowatt kour	1.45 cents.

There are, unfortunately, no figures yet for the summer months, but the yearly average will not differ greatly from the above as the apparatus previously in use, supplied nearly enough exhaust steam for heating the building. This can be readily seen from the large amount of coal required per electrical kilowatt hour in the heating season.

It is of interest to note that the amount of the electric light bill for the six months of '97-'98 corresponding to those listed was greater than the total present cost of light, heat and power.

Mr. J. Little, of 15th Street and 6th Avenue, has put his figures in my hands and it is regrettable that no hourly record of current is kept. The number of lights used does not vary greatly from day to day in a building of this character, except for a decided drop in dead summer season. I include these figures, as they are obtained from a store, small compared with the one just mentioned.

Mr. Little has one hydraulic elevator and a large number of incandescent and arc lights, and as these lights are used for display advertising, the figure given me of 125 amperes average burning for 14 hours per day, may be taken as approximately correct, especially as I have partially verified the figures. I have reduced the above, 20% for safety against underestimating kilowatt hour cost. The total kilowatt hours on this basis is about 50,000 per year.

The increases caused by electric light plant were

Coal, 320 tons, at $2.55	$815
Labor	824
Incidentals, including water	230
Total increase	$1,869
Interest and depreciation	250
Cost of electricity	$2,119
Cost per kilowatt hour less than	4.1 cents.
Coal per kilowatt hour	12.8 pounds.
Cost of coal per kilowatt hour	1.6 cents.

Mr. Little is considering the advisability of a large sign, and if he installs one, the additional cost will be merely that of the extra coal and sundries and additional wear and tear on machinery.

In the office building the load line is entirely different from the hotel or store, in that there is no load to speak of after 7.30 o'clock, while the hotel load is at its maximum at that time and the load peak is not so pronounced.

In the Hartford building, corner of 18th street and Broadway, a most careful and complete system of records is kept by Mr. E. Fred, the chief engineer, under the direction of Mr. Stephen Tyng, the agent in charge. Account is kept of everything in this building, even to the number of minutes the elevators are in actual operation, and through the courtesy of these gentlemen I am able to accurately determine the cost of electricity in this building.

Fig. 9 shows curves of daily load for each month in the year, the lower being the general average for bright days, the next, the general average for dark days, and the highest, the day with the maximum peak for the month.

The total *lamp* hours of light in one year were	2,000,456
The total *horse* power hours of power in one year were	77,869
The total H. P. hours light and power	211,233
Total amount of coal used	2,025,700 pounds.
Coal per E. H. P. hour	9.6 pounds.
The *total* coal per E. H. P. hour in summer was	9.4 pounds.
The *total* coal per E. H. P. hour in winter was	11.7 pounds.

To obtain the cost of electric light and power as in hotels, the cost of heating and operating this building without reference to light and power, must first be determined.

The coal for heating such a building would be about 210 tons a year. To retain the same class of service as at present, one engineer could be dispensed with. This leaves 1,600,000 pounds of coal chargeable to the electric plant and one engineer to which must be added oil, waste, etc., Edison and fixed charges.

The cost of electricity in this building is

Coal	$2,150
Labor	950
Lamps	200
Sundry supplies and ash cartage	400
Interest and depreciation	1,500
Edison charge (about)	1,800
	$7,000

The cost per K. W. hour	4.87 cents.
The coal per K. W. hour	9.4 pounds net.

The cost on central service would be 9.5 cents per K. W. hour for light, and 4.5 cents per H. P. hour for power, and the total $12,000 for light and power, a balance in favor of the isolated plant of $5,000 per year above interest and depreciation.

The use of a storage battery as an equalizer and to take the night load would reduce the total above, to about $6,250 per year, reducing the cost per K. W. hour to 3.9 cents. These figures include interest and depreciation charges.

It is interesting to note *en passant*, that the amount of time the elevators are actually using power varies from one-fifth to one sixth the total hours in commission and that the power used in the fields (five-sixths of which is wasted) amounts to about twice the power used to operate the cars.

The next buildings to be considered are those termed loft buildings, a class which in the modern form is a direct consequence of the tendency toward concentration, and is of rapidly increasing importance. These buildings which are beginning to cañonize upper Broadway (if one may use the term) as the office buildings have the lower portion, are from eight to twelve stories high, and from 100 to 200 feet deep, with their lower floors devoted exclusively to show-rooms and their upper floors to show-rooms combined with manufacturing. They are invariably of fire-proof construction, and on account of the small comparative cost of insurance and the large rentable space for a given ground area, besides the greatly increased conveniences, have practically driven the older five and six story buildings out of business.

These buildings require electric power for elevators and on the various floors for motors, used to drive sewing machines, presses and similar apparatus. Electric light is not generally used in great quantity in the upper portion, but in the basement and first two or three floors, a considerable amount is used for decorative effects, and lighting made necessary by the height of surrounding buildings. In addition to these demands there is another demand that has heretofore been met by gas, *i. e.* direct high temperature heat applications, for such purposes as irons, presses, ruffling irons, etc. With current at central station prices and the present keen competition in business, electricity cannot compete with gas, but to the isolated plant, the steady load of the electric heater is a welcome addition, which proves sufficiently profitable to make it worth while, even at a cost equal to gas. I installed a plant in such a loft building about eight months ago, and have received daily reports since the commencement of operation. The results are of interest, as they deal with a field that has been claimed by the central station men, as their very own, on account of the light average load factor of electric elevators and the comparatively short duration of the lighting load, as compared with that of hotels or office buildings.

This building is a ten-story loft building, 55 x 200 feet, having two passenger and two freight elevators (all electric) and an electric pump.

The 10th, 8th, and 7th floors are used exclusively for manufacturing. The 9th and 3rd are divided between manufacturing and salesrooms and the remainder are all salesrooms, storage and shipping departments. The first two floors are beautifully lighted, by over 500 incandescents, and previous to the introduction of the plant, none of the other floors was lit electrically, except in a few salesrooms. Two five-H. P. and one three-H. P. motors and a half-dozen fans constituted the whole electrical equipment of the tenants. Notwithstanding the paucity of electrical apparatus, the cost of electric power (including elevators) was $2500 per year and the electric light cost a thousand dollars more, though the care with which it was used may be seen from the fact, that no tenant was able to get the discount allowed for more than "one hour burning per day per lamp installed." In addition to this the gas bills amounted to about $700, principally for heating, and a small boiler was run on the top floor for dyeing, and running a small engine.

The building is exposed on the north above the fourth floor, and steam from a 150 H. P. Bigelow boiler supplied heat during seven and a half months. One man fired the boiler and half looked after the elevators.

The cost of heating the building and supervision (exclusive of elevator repairs, and other incidentals not germane to this subject) was $1600 per year (average).

The total cost of light, heat and power was about $5800 per year.

In October the electric plant was started, but on account of existing contracts, etc., and troubles incident to a change from 120 to 240 volts, the plant had not settled down to regular work before January.

The plant consists of three 33 K. W. direct-connected sets, supplying light and power from the same bus-bars at 240 volts; as there was only room for the old boiler, a breakdown connection was arranged with the street. The meter system was abolished, as one of the objects was to make the tenants anxious to stay in the building, and it was desirable that all possible friction should be avoided; they were therefore allowed to use unlimited light for the amount of their previous bills. Electricity was introduced in place of gas for heating irons and presses, and over 500 additional lights were installed, the additional light and electricity for heating being supplied at the cost of gas. Steam was supplied to the top floor for feather steaming, dye vats and engine.

The electricity now used by the tenants is about four times that used previously to the plant installation, and that the benefits are appreciated is shown by the fact that every tenant has signed a contract for two and a half years.

The cost of electricity in this building varies from 1.75 cents per K. W. hour in heating weather, to 2.07 cents per K. W. hour in non-heating weather, exclusive of fixed charges, which amount to $\frac{6}{10}$ of a cent additional.

The total K. W. hours in January were 11,550, in April, 10,250, of which about 2,500 K. W. hours per month are due to elevators.

The total coal used per K. W. hour in January was 13.6 pounds.

The total coal used per K. W. hour in April, 11.6 pounds. The increase due to heating, about 15 per cent. If the coal previously used for heating is deducted, the additional coal per K. W. hour, *i. e.* the coal used for electricity, was about 6.8 pounds in January and 8.7 pounds in April.

The coal used in May per K. W. hour was about 11 lbs. If an equal amount of electricity to that at present supplied were purchased, the cost on the central station service, would be $8200.

The balance in favor of isolated plant, on basis of equal amount of electricity supplied, is over $4000 per year in this case.

This balance would be increased by the installation of a storage battery, costing about $5000, avoiding the necessity of Edison service and by furnishing absolutely steady load to the engine, would reduce the coal used by nearly twenty per cent. The night service, which would be furnished by the battery, is of such small moment that the coal used for its supply would hardly be calculable and would certainly not exceed a few tons. The ten per cent. fixed charges on the battery would be overbalanced by the present cost of Edison service, and the coal saved would be net gain.

I have entered into the details of this plant to a perhaps unwarranted extent, but I think several important facts are brought out clearly.

First:—An isolated plant can be operated at a profit at rates lower than those of the central station, even on a wildly fluctuating elevator load, and a light load of short duration.

Second:—That the isolated plant service is a benefit to both tenant and owner.

Third:—Through the isolated plant, and only by this means can the use of electric heating appliances be brought to its full development.

Fourth:—The central station cannot compete, in so far as cost is concerned, with the isolated plant, where skilled attendance is otherwise required, and steam heat is necessary.

I shall have more to say on this fourth point later.

Curves are shown in Fig. 10 of the general day load of steady light and power in this plant, and in Fig. 6 a curve from five-second ammeter readings of lights and elevators is typical of this class of service. Figs. 7 and 5 show daily loads for months of January and April. On the same page is shown the total load of a combined loft and office building which is spoken of later on.

The loft building is possessed of one advantage, which in a measure compensates for the extreme irregularity of the elevator load. The electricity used in summer for lighting, *i. e.* the non-heating season, is extremely light when compared to the winter load, so that the amount of steam and water wasted is not so large, proportionately, as it is in a hotel plant where the demands are more evenly divided between the seasons. This condition is largely modified by motor and heating load.

I am unable to present complete figures on a small office building, as I have not found one in which hourly record of lights burning is kept.

The coal, oil and other items have been carefully noted in several, and as the number of lights burning in a small building does not vary greatly from hour to hour, or day to day, I will include the figures from one.

This building is thirteen stories, 50 x 30, and contains two drum elevators, two engines belted to 40 K. W. generators and one H. T. boiler. Night service is supplied by New York Light, Heat and Power Co.

The coal used varies from 1863 pounds in summer, to 2007 pounds in winter per day.

One engineer and one fireman run the plant, and the cost of heat, light and power, exclusive of night lighting and power, is less than $2500 per year.

Labor	$1250
Coal	750
Water	50
Oil, waste, lamps, sundries and ashes	250
Repairs (estimated)	100
	$2400

The cost of electricity in this building is ($2350 plus central station charge of $180 plus $500 interest and depreciation) less (labor $624 plus coal for heating, $380) = $2,025.00.

I have been in this building many times at all hours during the day and have never seen the ammeter register below 35 amperes on one side of the system. The engineer, who has been there several years, states the average lighting during the day to be 120 lights, and the maximum 350. It is therefore, perfectly safe to assume that the average current is 35 amperes and the yearly kilowatts 22,000. The power required by the two elevators is not less than 18,000 kilowatt hours per year, and the total power at least 40,000 kilowatt hours per year. This makes the cost per kilowatt hour, including interest and depreciation, 5.06 cents, excluding interest and depreciation, 3.81 cents. The coal per kilowatt hour is 7.25 pounds and the cost of coal per kilowatt hour is 0.925 cents.

These figures are, of course, approximate, but are correct within 10 or 15 per cent.

The cost of operating buildings without electric plants is even more interesting than the cost of operation where the plant has been installed.

I have noted in each instance, the cost at the present rates, of electricity in equal amount to that supplied by the plant, but the figures of the defunct Wool Exchange and of the Sohmer building, one containing a million and a half cubic feet and the other half a million, will confirm these estimates. Neither of these buildings has electric plants but each has two electric passenger elevators and one electric freight elevator.

In the Wool Exchange, the Edison company supplied 822.063 -16 C. P. equivalents of light, 38,375 H. P. hours of power, at a total cost of $7,700. Under the new rates, this cost would be reduced to $6,000 total, or as 70,000 K. W. hours were supplied, about 8.6 cents average price per K. W. hour. The cost of heating, operation of pumps and supervision (one engineer and one fireman, six months,) was $3,100. The cost, therefore, at present rates would be over $9,000.

Contrast this with the loft building cited, where, with the same cubic contents and over 100,000 K. W. hours supplied annually, the total cost is less than one half the above.

Although it is customary to speak of the elevator load as of comparatively small importance, in the Sohmer building it amounts to two-thirds of the total electricity supplied to the building.

In the loft building, on account of the electric heating apparatus and motors, it amounts to about one-fourth of the total quantity supplied. In many of the office buildings, it exceeds the lighting load, so that it is not a factor by any means despicable. It has one advantage to compensate for its momentary fluctuating character, which is, that owing to the fact that the bulk of weight travels downward at night, its requirements are not so severe when the period of heavy lighting commences as

TABLE.

SHOWING COSTS AND COAL PER K. W. HOUR.

Type of Building.	Type of plant.	COST PER K. W. HOUR.		Coal per K.W. hour.	Cost coal per K. W. hour.	Kind of load.	Approximate annual total K.W. hours.	APPARATUS SUPPLIED.
		Including Int. and Dep.	Excluding Int. and Dep.					
Large Hotel......	Compound	1.66 cents	1.30 cents	3.4 lbs.	0.50 cents.	Fluctuating.	825000	11 Elevators, Kitchens, Refrigerating and Laundry Machinery, Ventilating and Heating and Lighting, Pump.
Small Hotel......	Simple	2.45 "	1.94 "	8.37 "	1.17 "	Steady.	100000	2 Elevators, Kitchens, Refrigerating and Heating, Pump.
Apartments	"	4.70 "	3.80 "	7.00 "	0.88 "	"	55000	3 " " Lighting " " "
Department Store.	"	2.85 "	2.35 "	9.25 "	1.45 "	"	200000	4 " Cash Blowers, Laundry, Lighting & Heating, Pump
Small Store.......	"	4.10 "	3.60 "	12.80 "	1.60 "	"	50000	1 " Lights.
Large office bldg...	"	4.37 "	3.42 "	9.40 "	1.36 "	Fluctuating.	158000	4 " " Ventilating, Heating.
Small " "	"	5.06 "	3.81 "	7.25 "	0.93 "	"	40000	2 " " Heating.
Loft building.....	"	2.60 "	2.00 "	8.00 "Appr	1.02 "	"	100000	4 " Engine, Lights, Heating, Pump, Motors. Dye Vats, El. Heaters.
Loft and office....	Edison	Power 6.66 cents Light 17.20 "	....	....	....	....	30000 15000	3 Elevators, Lights, Pumps, Heating.
Loft building....	"	8.6 "	....	....	....	....	70000	3 " "

they are in the morning hours, and about the same amount of power is required from hour to hour and day to day.

In the Sohmer building, where the lighting factor would seem negligible, due to its exposed condition and small depth (30 feet), the cost of electric power for two passenger and one freight elevators, and electricity for lighting is over $3,900. The cost of heating and supervision is over $1,200—a total of more than $5,100, or $700 a year more than a building nearly four times its size, containing an isolated plant.

The total energy supplied in this instance amounts to over 45,000 K. W. hours, 30,000 for power for elevators and pump, and 15,000 for lights. The cost per K. W. hour for light was 17.2 cents, and for power, 6⅔ cents.

I am now installing an electric plant in this building, with storage battery auxiliary to take the fluctuation and night load. The total cost will be under $10,000, and the reduction in annual expense will certainly exceed $2,500—an excellent return on the investment. The cost per K. W. hour should not exceed four cents, everything included.

This building is 30 x 100 by 13 stories, and is exposed on all sides, so it may be considered to offer the most advantages to the central station, with the exception, of course, of the small private houses and small loft buildings, where, without a block system, no rival can enter. The curve of elevator and light load on a bright day shows the character of the service.

As an example of the large part that steam heating takes in the expense of operating a building, the Franklin building on Murray street is of interest.

During the months of November and May the coal per K. W. hour for electricity delivered was little over seven pounds, but during the months of December, January, February and March the coal averaged over 20 pounds. Evidently, this difference was not due to the quantity of current delivered, and in fact, the coal used would have been required whether the plant was in operation or not.

I have not given any figures from very small buildings, as the question of cost seldom enters: other items, such as convenience and space, are more important and decisive.

It would appear from these figures that the central station had no mission to fulfil, except that of supplying breakdown service and service to small buildings, and it seems hard to reconcile this with the enormous annual increase in the gross receipts and net returns of the central stations.

While it is true that a properly designed plant will, under good management, furnish current in a building of fair size at a cost of from 2 to 4.5 cents K. W. hour, everything included, it is not always advisable to install a plant. In many clubs, restaurants and other places, where quiet is absolutely essential, or where space is limited, or valuable, a saving of a few thousand dollars is not of sufficient moment to make it worth while.

There is another large class of buildings where the central station has a practical monopoly of the field at the present. This class is the loft or combination loft and office building, erected purely for speculative purposes. I do not mean by this, built cheaply, but for sale. Typical examples of these buildings are found along Broadway, and are similar to the loft building cited in this paper. These buildings are seldom more than half full for the first year or two, and are generally sold before the third year expires. Light or power are never furnished free. The elevators, on account of the comparatively few tenants, are inexpensive to run on the central service. The object of the speculative builder is to build the best building possible for the least money, and to keep down his operating cost as low as possible. In selling his building a lump sum is fixed, based on the estimated cost partially, but to a great extent on the operating cost, and by getting his current from the central station for elevators, and employing a cheap man to run a heating boiler and look after the elevators and machinery, the builder obtains a low operating cost and avoids an additional initial expense.

This class of building is comparatively new, and the possibilities of returns from the tenants have not been fully appreciated, but I have little doubt that a few more examples, such as the building cited in my paper, would convince the investor and through him the builder, of the advantage of installing an electric plant, thus reducing the operating cost by the receipts from tenants, but up to the present time this has not been the custom.

Personally, I do not believe that the central station receipts will continue to increase at the present rate, unless a supply of steam for heating, as well as electricity for power, can be furnished. It is a noteworthy fact that a great number of the ~~largest~~ office buildings down town are buying steam from a central station to run electric light engines and pumps, using the exhaust steam for heating.

The advantage that the steam supply company has over the electrical supply company lies in the fact that the former sells steam at the cost of coal plus labor, which in all office buildings is a large proportion of the total cost. The smaller the building the larger the proportion the labor bears to the rest. The electrical company cannot, on the other hand, hope to sell its electricity for much more than the cost of the extra coal, oil, waste, etc., and interest and depreciation charges on the plant, that would be necessary for local supply. The labor, is in all but very few cases, made necessary with or without the plant, by the need of skilled attention for elevator and machinery.

The cost of electricity in the isolated plant has been shown to vary from two to five cents per K. W. hour. A study of the clear and well written report of tne Edison Electric Illuminating Company of New York, is well worth while. From the figures given in this report, I find that 22,777,000 K. W. hours were delivered last year, at an average price of 10.6 cents. The actual cost to the Edison company (that is operating expenses divided by K. W. hours supplied) is about one half this sum. This latter, of course, does not include interest or depreciation on the investment. The average price therefor, at the present time, is from two to four times that of the isolated plant, and the cost of manufacture from one to two times the cost in the isolated plant, and the reason is not far to seek.

The isolated plant is never installed unless the conditions ensure comparatively large average use of the current and small additional trouble. On the other hand, the central station is obliged under the terms of its charter, to supply anybody within a hundred feet of its mains and the immediate consequence of this is, a large amount of unprofitable business. The average load for every hour of the twenty-four for the whole year, is about 2,600 K. W., while the maximum is nearly 12,800, requiring a large excess capacity over that generally used. As a matter of fact, the total capacity of the system in kilowatts is stated by the report to be over seven times the above average. As this excess capacity extends to the mains, as well as the generating apparatus, the proportion of fixed charges to cost of operation must always be enormous, although the storage battery will, to a certain extent, help them out.

I have little doubt that a continual decrease in the prices will be made by the central station, as their business increases, but a similar decrease is going on in the isolated plant, both from the increase in knowledge of the requirements, and the increase in the uses of electrical apparatus, uses which are largely stimulated by the cheap supply. I also believe that there are very few plants, that would not welcome the central supply of steam.

A continued erection of large buildings in a congested area will inure to the advantage of the central steam supply and for the reasons above given, to the disadvantage of the central electrical supply.

Only one office building in ten, of modern construction, takes its current from the central station, and with the large loft buildings, the story will be the same, as soon as these buildings pass into the hands of investors.

The department stores, hotels, large restaurants and apartment houses (where light is given free) have already found the isolated plant to be the cheaper and the continued disappearance of small buildings will increase these conditions, unless the central station companies can discover elevators that will take care of themselves, or incorporate with the supply of electricity, a cheap supply of steam for heating.

DISCUSSION.

THE PRESIDENT:—The paper is now before the INSTITUTE for discussion.

MR. H. WARD LEONARD:—This subject of the isolated plant is a very old one and has a good many points in it pro and con. There has been no very radical change in the cost of production of energy in isolated plants over quite a period. I remember about twelve years ago I secured a great abundance of statistics covering probably three hundred isolated plants in the west, and although the cost varied, as, of course, such figures always will, the statistics were based upon answers to a very large number of questions that were framed with the idea of getting accurate figures, and the owners in those days were I think perhaps a little more interested in the results than they are now; at any rate they very gladly went to considerable trouble to give me the figures. I remember that the cost at that time, that seemed to be attainable, in a well installed plant was about a quarter of a cent for 75 watt hours, which would be in the neighborhood of about 3⅓ cents per kilowatt hour, and it seems to me that about 3 cents per kilowatt hour is pretty close to what can be done in an isolated plant to-day if the same consideration be given to the factors of the cost that I gave at that time. Personally I believe that an allowance of five per cent. for depreciation is insufficient.

While you can borrow money at five per cent. you can't borrow money at five per cent. on the ordinary isolated electric light plant, and if you are going to invest money in an electric lighting business as a speculation you would not be apt to invest your money when the maximum return possible would be five per cent., in view of the risk. My own opinion would be that 15 per cent. would not be at all too much to allow for the interest and depreciation charge on a modern isolated plant.

Another point which has always struck me as of very great importance and which is not sufficiently allowed for, is the value of the space that the plant necessarily occupies. While it may be true that in some instances plants will be found where there is no other use for the space, I don't think that is the rule by any means and I remember that without considering at all the question of space for a storage battery I have frequently been much puzzled to find room enough for the electric plant and it is very often a very serious problem. In the modern tall office buildings the space that the plant occupies in the basement is an important matter, and although the owner may not appreciate it fully, I consider that a continual charge should be made against the operating cost, for the maintenance of that plant in that space, because if he had not used the plant in that space he would have used something else there. So that it seems to me that with an allowance of ten per cent. only for interest and depreciation, and in disregarding entirely the value of the space occupied, the present paper leans too strongly in favor of the isolated plant. Of course, it is incontestable, at least I think so, that under favorable circumstances a large isolated plant can produce electricity at a price such that the central station company would have difficulty to compete with it, but I do believe that a great many isolated plants are being operated at a higher cost than the central station company could afford to take for the service. In some of the cases here quoted, even with this rather small allowance for interest and depreciation, and no allowance for the value of the space occupied, you will find 4.7, 4.3, 8.06, per K. W. hour, and figures of that character. I believe the central station would provide electric energy at those figures if they saw what the load was. These curves are particularly favorable as compared with the average curve of the central station, and the load factor being high, a low rate could be made for such service. I have always felt that there was quite a field in the direction of supplying energy from the central station at a flat rate per kilowatt for a 24-hour service, allowing the customer to have his own storage battery and thus purchase his power at the minimum rate. But as the author stated, there are many instances in which the economies which might be derived from a storage battery are not realizable because of the lack of space, and I also feel that a person who installs a storage battery in connection with an isolated plant necessarily adds or should add almost a dollar a day to the value of the expert labor if he is going to keep his plant to the degree of minimum maintenance that he had before he put the storage battery in. And I personally think that the percentage allowance for depreciation,on the storage battery even to-day, is not such as compares favorably with that of a plant that has no battery.

It seems to me that the whole question boils down to the value of the heat of the exhaust steam as against the economy of production on a very large scale by a central station, over a small plant. Where the value of this heat is large the isolated plant would necessarily have an advantage. But certainly a central station plant should be able to, and I believe it does, produce a kilowatt hour at a very much lower rate than it could be produced by an isolated plant if we disregard the exhaust steam. Frequently figures of this character are largely a question of methods at arriving at results but there is every reason to expect that under skilled guidance a large central station should be able to produce energy at a rate such that nothing could compete with it except for this factor of the value of the heat for the exhaust steam. And since instances seem to be developing where the owner of an isolated plant finds that he can purchase from a central station, steam to operate an isolated plant, cheaper than he can operate his plant by himself, it would look as though in an average case the question of cost would be in favor of the central station, provided that the central station were able to sell its current for a twenty-four hour service and consequently sell it at a low rate. It seems to me that the central station ought to be willing, and I presume they are in some instances willing, to sell a steady load at about 3½ cents per hour for annual service, and I believe that energy bought at such rates and stored and used as required will compete quite favorably with the average isolated plant case.

But of course this question of elevators is a factor that nearly always is an important one, and the introduction of any means of equalization, such as a storage battery, would I believe operate elevators at an advantage as compared with the rates which would have to be charged by the central station if the current were so excessive at times, as the direct load of the average elevator demands, when operated by the ordinary methods in use to-day.

It seems to me that in this kind of a case the greatest saving can be effected by the owner making use of the service of a thoroughly competent and unbiased engineer, because each problem is one which must be studied by itself, and the various factors of electric light, heating, elevator service, etc., vary so widely that what in one case might result in a saving would in another case be a serious disadvantage.

Mr. Arthur Williams:—Nothing, perhaps, could more clearly illustrate a point I wish to make in regard to the use of the exhaust steam of a private plant for heating than the curve which Mr. Moses has drawn—the characteristic load curve of an office building. During the early part of the day, and in the early morning hours especially, the need for heat is at a maximum, whereas there is very little demand for artificial light in the late afternoon, when the demand for lighting, and therefore, the supply of exhaust steam is at a maximum; the necessity for heating is at a minimum.

Mr Moses' paper is based almost entirely upon conditions as he understands them in New York City, where he believes the possibilities of the near future for the central station are relegated to the supply of current for small customers and breakdown connections for isolated plants. The field of comparison selected is so local that I feel you will justify a reply, also local. I did not succeed in getting a copy of the paper until just before leaving New York Saturday and am not able, therefore, to make a statement in regard to each of the buildings to which it refers, but a number may be identified, and having been connected closely with that branch of the New York Edison company, I am able to show, to some extent, the side that has not here been presented. I learn in answer to a telegraphed inquiry, that of 118 large buildings erected in New York City in 1898, in the plans of which the isolated plant for both lighting and power was a factor, all but 22 decided to use the Edison service; of 66 such buildings erected in the first half of 1899, 60 have decided to use the Edison service.

There were several buildings in connection with which we submitted figures of cost, where in view of the low average use and the coincident local and system maxima, we felt that even at best prices the very large investment which the company would be compelled to make would leave us at the end of the year a considerable loser, and, while we desire to have it understood that our company is prepared to supply electricity in any building for any of the established uses, we did not regret the conclusions of the engineers and owners. It must be apparent, however, that in an instance where the company finds that its established rates subject it to loss, the conclusions of the engineer who encourages his principal to make very large and costly investments in generating and other electric machinery must be subject to serious question.

While attending the second electrical show in New York, an agent, in attempting to sell me an isolated plant, stated that the cost of operation was approximately half the cost of central station service supplied in New York. I said as central station service costs approximately one cent a 16-candle lamp hour, that of his plant would cost approximately one-half cent, to which he answered in the affirmative; I then asked him that if the central station service cost only one-half cent—an amount much in excess of the average price paid by the large buildings in New York—could the plant be operated at one-fourth cent. To this he made no reply, feeling, evidently, that he was getting into deep water.

The same motive that, in many instances, leads a consulting engineer to recommend the private plant actuates the operating engineer in keeping it in operation. An instance occured not long since in which the engineer of a large down-town building stated to the committee in charge, who were considering a proposition we had submitted to them, that he was making electricity at "one cent per K. W. hour or a little less," upon which statement, and without further investigation, the committee decided that it could not afford to substitute the local Edison service at five cents per kilowatt hour. It should be said that our own estimates, most carefully and, we believe, most accurately, prepared, showed a very large percentage of saving, with much greater satisfaction, in abandoning the plant.

Our experience with a large club taking central station supply, might be interesting: We were receiving approximately $8,000 a year from an installation of 1,200 lamps. The house committee felt that this was more than the club should pay, and authorized one of the prominent engineers of New York to prepare the necessary plans and specifications, upon which bids were obtained. These bids for the electrical and mechanical plant amounted to $35,000, to which the architects thought $10,000 should, or might, be added for the necessary building alterations. The engineer had stated to the committee that when installed, this plant would save enough each year to repay its cost in two or three years, after which the club would get its light for nothing, and this statement was repeated in all seriousness by one of the members of the committee at another meeting which I attended. It took little consideration on the part of the committee to find that far from saving anything by installing a plant they would in all probability increase their annual charges by as much more as they were then paying the Edison company; the fixed charges alone, including interest, depreciation, taxation, insurance, and "up-keep" expenses, amounted to very nearly 20 per cent. of the investment, or approximately $7,000 and they had been paying us only at the rate of $8,000 for everything.

One can hardly disagree with Mr. Leonard that the author's 10 per cent. is not enough for the fixed charges of a plant. Our New York practice is to include the interest value of the money invested at about 6 per cent.—it is worth much more when taken out of one's business—the depreciation at 8 per cent. allowing about twelve years' efficient life of the plant, taxes at 2 per cent. the actual rate being about 3 per cent. and 1½ per cent. for insurance, this item on steam and electrical machinery, being subject to variation. It will be seen that a very small addition for "up-keep" repairs, rent, accident insurance, or any of the other many items usually forgotten but still incidental to plant service, would bring the annual fixed cost to 20 per cent. of the first, or installation cost.

The statement of the writer, that the expense of operating a private plant is the difference between the cost of the mechanical service of the building before and after its installation, should be qualified. And if the costs are given on the kilowatt basis, owing to the tendency to over rate plant output, this statement or definition could become very misleading. The tendency, even in our own stations, where our people have no possible incentive, is to read the indicating instruments on the high rather than on the low side, and the average plant engineer, desiring to support and increase the importance of his position in every possible way, is certainly as cautious in this direction. In addition, the tendency of all consumers where not under the moral influence of the meter, is toward a very wasteful use of light or power.

Not long since, while the family were at dinner, we found three electric lights burning in the small parlor of an up-town apartment supplied by a private plant and the use of light in the halls and elsewhere seemed equally extravagant; and I recall an instance in which by placing a meter on an apartment where light was included in the rent, it was found that the cost of current used wastefully and for ornamental, in addition to the proper purposes, amounted to more than the landlord received from the tenant. It is safe to say that in either instance, and without reference to the source of supply, a meter would have cut down the use of current fully one-half, if not more—in fact in the second instance, when the tenant learned that the landlord had a meter on his circuit, though even then not paying directly for light, the consumption became about normal, which was less than one-fifth the previous consumption.

Another instance in our experience, showing that the tendency is to overstate private plant output, arose in connection with a large building for the supply of which the owner had asked our annual costs on the basis of the number of K.W. hours reported by his engineer. On finding that our first estimates greatly exceeded the cost of running the plant, we were led to make an investigation, which revealed the fact that the ampere meters were reading almost double the current generated, and that several items of expense properly charged to the generating plant had been charged to other parts of the building. The true figures of consumption showed that the central station supply offered a large saving, and the plant has since been idle. Many additional instances could be cited, showing conclusively that the K.W. consumption of a building is a very unsafe factor to determine the relative cost of central station vs. private plant supply.

For the large hotel of which Mr. Moses speaks, the Manhattan, of New York, the annual cost of electric light is placed at $13,000. These costs are estimated deductively by determining the difference in the cost of fuel and other supplies during a period in which the hotel was supplied with current from the local dynamos and for one in which the local Edison service—for lighting only—was used. It should be remembered that the building has ten or twelve electric elevators, which, owing to the difference of voltage required, compelled operating the plant, notwithstanding our station supply of light during the entire period of the test.

That the sum given does not fairly represent the real cost of electric light is readily shown by the fact that the hotel last year consumed 6,000 tons of coal, costing $20,000, of which only 1,400 tons, costing $4,500, is allowed for light. Two thirds of the coal consumption, or 4,000 tons, it must be evident, are to be charged to both lighting and elevators; the elevators alone, therefore, if the statements made are accurate, would have consumed 2,600 tons, when certainly much the "smaller half" of the power generated was required for that purpose. The only fair test of this character would be to shut down the entire plant, supplying elevators and auxiliary motors, as well as lights, from the Edison service, crediting to that service in a comparative estimate the resultant saving that would be accomplished in labor, fuel, supplies, etc.; the space occupied could be utilized for other purposes and doubtless has a high rental value. The fact is that instead of costing $13,000 annually, the lighting of this building is probably costing more nearly $20,000, or, including the elevators, more nearly $30,000, whereas our estimate of Edison service for all purposes has never exceeded $25,000, and, under proper control, I doubt if the cost would reach that amount.

The apartment house mentioned by Mr. Moses offers another instance in which instead of showing a saving through plant operation we believe that there would be a very considerable advantage to the owners by adopting Edison service. I have no estimate of the present cost of running this building, but, taking another building, a hotel very much like it as to size, but where the conditions of service are much more onerous, the bills for the entire supply last year were something under $3,000. The proprietors recently considered the installation of a plant, but on investigation became satisfied that their existing service could not be replaced at a cost much below $6,000 annually, which agrees with the figures given by Mr. Moses amounting to about $7,000 for the apartment house in question. Allowing a little more fairly for labor and taking a more conservative view of the investment value, even this high relative cost for the apartment house becomes nearly $10,000, undoubtedly a more accurate estimate.

The electric elevator supplied by meter is the cheapest service a central station can give. The average cost of 15,000 H.P. supplied by our company last year was $18 per H.P. installed or approximately half the estimated cost of delivering coal at the doors of New York boilers. Only last week the proprietor of an apartment house found that his elevator service, continued through twenty-four hours daily, was costing approximately $3,800 per annum, whereas we have never supplied an elevator of that type at a cost exceeding $500, and the average is very much less. He has abandoned the use of his plant, and is now arranging to substitute an electric for the hydraulic elevator. When the change is perfected the lighting and elevator service together will hardly amount to the former costs of the elevator alone.

On the basis of the estimate Mr. Moses has given, the electric light and power supply of the office building referred to on page 336 costs $7,000 per annum. It will be observed, however, that only 10 per cent of the installation cost, or $1,500 is allowed as covering interest, depreciation, and I presume, insurance, taxes and rental, though these items are not mentioned, while 20 per cent., as has been shown, is more nearly on a not too conservative basis, the proper charge, increasing in this one item, the estimated cost by $1,500. Another indication that the estimate is insufficient is found in the item allowed for lamps; $200 represents 1,000 incandescent lamps at 20 cents each, and as the reported consumption for the year was in excess of two million lamp hours, the average life of each lamp would be in excess of 2,000 hours. It is questionable whether any plant, using efficient lamps, where the standards of light are fairly maintained, secures a life of more than 600 hours. Either the item understates the cost of lamps, or the reported consumption of current is largely overstated.

Four hundred dollars has been allowed for "sundry supplies and ash cartage," an item which must include the water supply which last year cost more than $600, and the oil supply which amounted to more than 650 gallons; and according to the estimate presented to us, one man, receiving $18 weekly, runs this plant of four high-speed engines and dynamos, the necessary controlling and regulating apparatus and a storage battery. The estimate given, by fairly allowing for items mentioned, could easily be increased to $10,000, and it is extremely doubtful whether even this item represents the cost to which the proprietors of the building are subjected; on the other hand, I am certain that, supplied from our station at the usual rates, the cost for both lighting and elevator service would not exceed $7,500 per annum.

A storage battery is considered a medium of saving, and the cost of making the current, which I presume means its delivery at the switchboard or at the battery, as might be desired, is placed at 4.37 cents per K. W. hour. Our storage battery prices in New York start at 6 cents per K. W. hour, and descend on a graduating scale to 3 cents, a price with which I have yet to know of any isolated plant competing, operating charges alone, but fairly, considered. But even at 3 cents we have found it very difficult to convince anyone of the economy of the storage battery for which the central station supply is certainly more regular and better assured than that of a plant.

The writer's reference to the New York Wool Exchange, and its comparison with a large loft building, (page 342) brings out clearly the point I have made as to the wasteful tendency of engineers and tenants where current is supplied without direct charge. The loft building with a plant consumed last year 100,000 K. W. hours, or 2,000,000 lamp hours; annually; the Exchange, having the same cubic contents, supplied through meter by the Edison company consumed only 822,000 lamp hours. The Exchange building has a comparatively high load factor while a loft building, such as that of which the author speaks, has the lowest load factor of any class we supply. As no complaint of an insufficient supply has been received from those in charge of the Exchange, we assume that they have had all the electric light and power required.

Allow me to draw your attention to the conclusion of the author regarding the Sohmer building of New York City. From his paper it is evident that the cost of electricity for lighting and power last year was $3,900, to supply which he now intends to install an isolated plant at a cost of $10,000, and expects to obtain current at about 4 cents a K. W. hour. The consumption for all purposes last year was 45,000 K. W. hours, which, at 4 cents, would cost $1,800. The fixed charges at 20% of the installation cost amount to $2,000, in addition to which it will be necessary to provide fuel, labor, incandescent lamps, engine room supplies, etc., as well as an auxilliary source of supply consisting of either gas or electric light service. Two men added to the labor force would at least add $1,500 to the fixed charges of $2,000, leaving a margin of $400 for coal and everything else required to replace the present service for the lighting and elevators.

The author's distinction between the practice of the speculative and the investment builder is to be taken, curiously, very much in the favor of the side he does not advocate. The speculative builder of New York erects the most substantial building that can be designed, usually with the expectation of filling it with tenants and of selling it at any time within two or three, or more years, basing his price at that time upon the investment value as determined by the net return which he is able to show as a result of the year's operation. It is an investment of the highest class, with every possible incentive on the side of the temporary owner to the highest economy—the far sighted economy. I do not remember at the moment of a single builder of that order who installed a plant last year; it was toward that class of builders that our first missionary efforts were directed, and their present practice is at extreme variance with that of three or four years ago, when for every such building a private plant was contemplated and usually installed.

Contrast this with a building erected by a permanent investor, in which a plant costing $35,000 was installed, to provide the electrical supply for about 6,000 incandescent lamps. The full service for both tenants and owner, who provided the public lighting, was supplied by the Edison company for something more than a year at an annual cost slightly under $6,000; 10 per cent. of the investment cost, the amount which the writer of the paper thinks should be allowed for fixed charges, is $3.500 and 20 per cent, which we think should be allowed, is $7,000 or, in itself, more than all the charges of the Edison company. The plant has been recently started, and I understand that the single item of labor requires the addition of four or six employes in the engine and boiler room. In its operation it is expected to make money and supply current to the tenants at less than the amount of their payments of the last year to our company.

The central station requires machinery for about 30 per cent of its connected installation and finds this provision ample. The isolated plant, if properly installed, must provide from 100 per cent. to 150 per cent.; at the first figure, you have three times the machinery, installed under much more expensive conditions, upon which to base the fixed charges. Then consider the number of employes. Our company requires a man and a half to operate a 2,500 H. P. engine, while the 800 private plants of New York, aggregating probably only half our supplying capacity, will average at least three men to each. I think the entire city south of 10th street is operated through the night with a force not exceeding a half-dozen men, a number not greatly in excess of the force that would be required in one of the large buildings were night as well as day service supplied.

The author suggests that a discussion on the subject of gas engine plants would be interesting. I recall two instances of a number that have arisen in our experience. The first one is of a restaurant of considerable size, in which a gas engine and dynamo combined, of the latest type, were surreptitiously installed—I say surreptitiously, because the proprietor feared to communicate with us, thinking we would oppose his plans. We, however, learned of the matter before actual operation, and, on going over the contracts, found that the guarantees as to the consumption of gas, on a minimum basis, placed their cost alone at $3,500 (gas at $1.15 per thousand feet), whereas our Edison bills had amounted to only $2,700 for the preceding twelve months. The plant has never operated for more than a test run, and is one of a number which I think could be purchased at a very low figure.

The second instance is of a large hotel, in which the dynamos were operated by gas engines. Question having arisen, we installed Thomson meters on the dynamo leads and found that in a month, in which the gas meter showed a consumption of $206, the electrical meter at our published rates, recorded only $204. Nothwithstanding recent reductions in the price of gas, I think all of the gas engine plants in the district lines of the New York Edison company can be counted on the fingers of the hand.

That the superiority and relative cheapness of the Edison service is becoming more and more highly appreciated by engineers, architects and owners, is shown by the number of plants that are abandoned, as well as by the relation existing between the plant and central station supply of the new large buildings, as indicated by the telegram I read a few moments ago. Several of the largest hotels in New York have plants idle in their cellars because of the saving in favor of Edison service, (and yet hotel service is the most difficult to compete with) and one of the largest down town printing houses also abandoned its plant something more than a year ago, finding that the Edison service, which has never failed them, costs no more than the direct operating charges of the plant service, which gave a great deal of trouble. I speak of operating charges only, in each of these instances, as in the abandonment of a plant for competitive service, fixed charges of all character must be ignored, the investment having been made and the plant once installed possessing very little intrinsic value.

MR. STEINMETZ:—There is one reference in the paper recurring a number of times, regarding the fluctuating load imposed by electric elevators, and the hope is expressed that storage batteries may be used to equalize this load.

There may be some doubts whether storage batteries are advantageous in such isolated plants or not. They undoubtedly have been a very great success in large low tension direct current systems, but just as undoubtedly they have been an entire failure in street car propulsion and similar applications.

It is true that the elevator load is a severely fluctuating load. I doubt, however, whether in installing electric elevators due care has always been taken to make this load as little fluctuating as possible. There must always remain the fluctuation between the power consumption when running, and the absence of power consumption when standing still. It follows thus that the problem of the electric elevator in its relation to the power supply system is to make the power consumption as uniform as feasible during the time the elevator is running.

This means first that the starting current of the elevator should not be greater, but rather less than the running current. This can be accomplished by the use of a shunt motor of fairly low magnetic density with a powerful series coil which in starting remains in circuit until all the starting rheostat is cut out.

Second, to use power as uniformly as possible when running makes it obviously objectionable to use gravity for the descent, since thereby the power consumption during ascent is doubled. On the contrary, the car should be overbalanced for the average load of the elevator so that at the average load the motor supplies only the work of friction, more work when ascending with heavy or descending with light load, and less work in the opposite case. While this consideration really appears self-evident, yet I doubt whether it is always taken into consideration when installing electric elevators.

MR. MOSES:—Replying to Mr. Leonard, I wish to emphasize the point made in the paper, that the cost of electricity in a building plant is in no ways comparable to the cost in electric lighting plants out west. In an electric lighting plant, labor, coal, rent, taxes and insurance, are directly and solely chargeable to the electricity. In a building plant on the other hand, only the labor additional to that required for supplying other functions of buildings and only such coal as is additional to that required for the heating and for other apparatus (which can be equally well supplied by exhaust steam) are properly chargeable to the electricity. Rent, taxes, insurance, depreciation and interest charges are also quite different in building and in isolated plants. This brings me to the question of interest and depreciation charges. My figure of 10 per cent. has been strongly objected to. I still adhere to it for the following reasons: while I would not advise any client to invest his money in an electric plant unless more than 10 per cent. clear was assured, yet I consider five per cent. or even four per cent. as ample to allow for interest charge, as this is in excess of the actual amount an investor or owner of a building would pay for the money required to replace the capital invested in plant. As regards depreciation charge, I allow an average of 5 per cent. per annum (which with interest at five per cent. will equal original capital in fifteen years), for depreciation on the whole installation, which includes boilers, piping, pumps, engines, dynamos, foundations, alterations to existing building, boiler setting and alteration to wiring. This allowance is excepted to, on the ground that no plants last fifteen years. I can cite dozens of existing plants where boilers have been in daily operation for over twenty years, and are still allowed to carry from sixty to eighty pounds pressure, and many engine plants have been running for the same length of time. Direct current electrical machinery has only recently been standardized, and as electrical building plants are dependent on the existence of large buildings they are, of course, of comparatively recent growth.

Judging from the construction of the engine and of the dynamo, the latter should last at least thirty years with but little repairs. I am safe therefore, I think, in considering fifteen years as the average life of a plant. It has been objected, also that the continual changing of design should involve a larger depreciation charge. This I do not think will bear investigation.

The investor can continue to produce current from the plant he installed at the rates stated in the paper. If large improvements are made in machinery, he may find it worth while to completely change his plant. In this case, however, the cost of producing current will evidently be very much lower, so that, though his depreciation charge may have increased, his cost of operation will have decreased in greater proportion, as in any other event the change would not have been made. The rental value of the space occupied by the plant has been dwelt upon. Generally this value is not calculable, as the greater part of the space used is required for elevator, pumps and other machinery, boilers and motors, and the space actually used by the electric plant is of no consequence. In some cases the added space is valuable, and must be taken into account. In general this is not so. Mr. Leonard states that the question depends on the comparative value of the heat obtained from exhaust steam vs. the increased efficiency of the large engines in the central stations. This is not the case. The value of the heat from the exhaust steam, while important, is not the deciding influence. A few of the reasons why the building plant can manufacture and deliver electricity cheaper than the central station, in addition to the benefits of use of exhaust steam for the heating, are that in the building plant little or no additional labor is necessitated by the addition of an electric plant, and such labor is always cheap. In the central station; labor for firing, for running engines, keeping books, collecting bills, inspecting outside and internal constructions testing meters, and selling current are direct charges on electricity. In the building plant there are no legal expenses, no treasurers or officials with large salaries, no increased insurance or taxes, no rent of subways (and generally no rent at all); besides which, interest and depreciation are reckoned on actual cost and only on the additional apparatus required for supplying electricity, and there are no patented distributing rights for which large sums are payable annually. In the central station all these items, which constitute more than two-thirds total cost are direct charges on the electrical current.

Mr. Williams intimates that because the Edison company installs generating apparatus and mains for but 30 per cent. of the total connected installation, that therefore, their investment is less, per unit of current delivered than in the isolated plant, where a capacity of from 100 to 150 per cent. total connected installation is installed. This is not the case. It is true that the capacity of the Edison stations is only 30 per cent. of the total connected installation, but the averge load is only one-seventh of this capacity. In this building plant, on the other hand, the average load is from one-half to one-quarter the total capacity installed. That is, the cost of the plant in the Edison stations for a given load is from two to three times the cost of the building plant. The fixed charges are at least twice those of the building plant. Replying to the criticisms on the use of the storage battery; the battery merely takes the fluctuating load, the integral of which is comparatively small. The allowance for loss in passage through the storage battery is about one-fifth of this and is not to be compared with the benefits derived from the steady load on the engine. Generally, however, its chief claim is based on the fact that it takes care of the night service.

With the storage battery in use on the Edison system the conditions are quite different, as in this case all the power and light are passed through the battery, and with the same percentage of loss the benefits from reduced rates are swallowed up in low efficiency. Besides, the cost of a battery to take the whole load is very many times that of one to take the fluctuation of the load only.

Mr. Williams states in his discussion of the figures from a large hotel "that it must be evident that two thirds of the coal used in this hotel is chargeable to the electricity" and has evidently misunderstood my meaning in the paper. The "1400 tons of coal costing $4,500" are all that are actually chargeable to light and power not to light alone. To anyone familiar with hotel work it must be patent that heating the building, cooking and running laundry and ice machinery use more than half the steam required in the building, and it is this amount of steam or coal that I have called "the constant requirement" and which I have approximated by taking periods with nearly constant conditions of weather and service in which equal amounts of electricity were supplied. By subtracting the coal used in the first period from that in the second period a difference was obtained, which, as the electricity supplied in each case was the same, was roughly that required for the other uses enumerated above. This difference divided by the difference in the number of days in the two periods would give the "constant requirement" per day. His suggestion to throw the entire plant of lights and elevators on Edison circuit for a test was impracticable as the elevators are wound for 120 volts. Mr. Williams also criticises my figures on the office building and brings up a figure of $600 for water supply which he assigns to the plant. Most of this is used by the tenants through the building for other purposes, but I charge water to the plant at the rate of 25 cts. per ton of coal burned, which is the equivalent of 8 lbs of water evaporated per pound of coal, making due allowance for the fact that in winter the water evaporated is returned nearly wholly to the boiler and reused. The "lamp" and "oil" terms are taken from the records and it is not therefore necessary to discuss them.

In the hotel spoken of by Mr. Williams where the plant has been shut down for one year and three months, the owner has stated to me that he reckons the increased cost of the Edison service at about $100 a month. This is about the actual money increase over what it cost them when they ran their own plant, but as about $2500 was paid for current from an outside source at retail rates, the cost of operation of the plant (present cost $6660 a year) was about $3000. The plant installed was too small for the work and the breakdown connection was a necessity. The owner does not care to invest the necessary money now for putting in a proper sized plant, as the old hotel business is, to a certain extent, uncertain. In the Grand Union Hotel, where I installed a plant, and where no breakdown service is used, the cost of operation this year, excluding all saving due to decreased cost of gas, is over $5500 less than it was last year. One-half of this saving was due to the use of small coal, but the remainder is the difference between the cost of electricity $4558 from the central station and the cost of electricity from the private plant. In reality, the saving effected was greater, as the hotel has increased its use of steam. The total cost of installation was $8000 and cost per K.W. hour, everything included 2.19 cents.

Mr. Williams refers to the moral influence of the meter and objects to my basing costs on the K.W. hour supplied. Where light and power are furnished free to the tenants the meter will have no moral effect. Where they pay for it themselves it has a decided effect. In one building of the same size as the Wool Exchange, cited in the paper, and with similiar conditions, current is sold to the tenants by meter at Edison rates and the profits pay the expenses of operating the whole building. In this building the cost per kilowatt hour is five cents.

Mr. Williams criticises my statement that in the Sohmer Building current at four cents per killowatt hour can be produced. I have taken up the question of interest and depreciation charges already, and therefore deduct $1000 from his estimated annual cost, and the remaining $1,500 which he charges to labor and extra coal, may properly be divided in half, as the labor charge, is not increased in any way. The coal will be increased by about $500 per year and $250 is sufficient to allow for oil, waste and supplies. That gives a cost of $1,750 to which should be added cost of repairs, of lamps, ash cartage, boiler insurance, extra water and contingencies, not more that $500 or a total annual cost of $2,250.

The kilowatt hours supplied, as stated, are 5,000 for light and 30,000 for power, exclusive of the light used and paid for directly by the tenants of the first seven floors. I am perfectly safe therefore, in assuming that the total kilowatt hours supplied will exceed 60,000 which at four cents gives $2,400—that is $150 more than the total arrived at above, making all allowances for contingencies. Mr. Williams cites figures of comparative number of new buildings erected with and without plants and I, of course, presume them to be correct. Such general figures are extremely misleading as is shown in this instance, for it must be a matter of common knowledge that no large office building, properly so-called is erected without its own plant, unless the controlling officer of the erecting company happens to be also strongly interested in the lighting company. The speculative buildings were frequently erected without plants, but where the building is 50′x200′ or larger and 10 or more stories high this practice is antiquated and the owners of the buildings already up are rapidly installing plants. No large hotel is erected without a plant and the only large buildings that do continue to use street current are apartment houses, were light is paid for by the tenants directly and old houses where the owners do not feel warranted in spending more than is absolutely necessary.

If the present methods in building plants and in central stations were those of five or three years ago, the answer as to the comparative value of the two services would be more difficult.

A few years ago all the trained engineers were employed by the central station or trolley companies, and every advantage of technical skill and large capital was on their side, while the building owner dependent on his stationary engineer fell rapidly behind in the march of improvement. Nowadays the owner of the large office, hotel or loft building has a first class progressive engineer in charge or employs a consulting engineer to keep a general control over the staff and operation. In some instances the complete plant is run for a fixed sum per year by a contractor who is able to make money himself and save the owner money as well, by purchasing large quantities of material at a time. This method while it renders many of the central station economies of both material and labor available to the isolated plant is only to be recommended when the contract is for such a length of time that the contractor will see that proper repairs and additions are made, whether he is momentarily financially the sufferer or not. It is these facts, that repairs are seldom made by the contractor until the plant breaks down, and that no improvements are possible on account of the short term of the contracts that have brought the system of contracting into serious disrepute. The same results can be obtained by placing the operation of a number of plants the purchase of their supplies and the employment of labor in the hands of a technical engineer, who should be paid a fixed sum per month by each owner. This will give the owner the benefit of the best engineering, improvements and all economies possible to large buyers, while at the same time the plant will not suffer through neglect of proper repairs.

MR. STEINMETZ:—In this paper, the depreciation is estimated on the basis of a 15 years life of the station. I like to ask the question whether any one of the gentlemen present knows of any station where apparatus and machinery installed 15 years ago are still in satisfactory operation. I need not ask the question whether you know of stations where apparatus installed less than five years ago has been thrown out as antiquated and unprofitable, since such stations are numerous.

MR. WILLIAMS:—That question is answered by Mr. Moses himself where he refers on page 326 to the plant of the Manhattan Hotel as being even now "slightly old-fashioned." This plant was installed in either 1896 or 1897 under the supervision of one of the most competent engineers in New York. And would you allow me to say, correctively, of the Broadway hotel mentioned, that the books were carefully gone over, and there can be no question as to the figures which have been given.

Of the large office building, the point made was that the entire expense of lighting and power service supplied from the Edison station did not substantially exceed $5,000, while the fixed charges on the investment alone were fully $7,000, again omitting the question of rent, which in that neighborhood is very considerable. In our discussion one side of the question has been entirely overlooked, the extent to which provision for a large mechanical plant increases the cost of constructing the building. In the Manhattan Hotel, I have not the slightest doubt, the necessity for providing for the engine and boiler-room in connection with the foundations and building structure, increased the cost fully $50,000, incidental to which there are fixed charges which must be considered in one way or another, an expense of the hotel.

The storage battery losses, as Mr. Moses says, are confined to but a percentage of the total output, but on the other hand this percentage must bear all fixed charges of the battery.

MR. C. W. RICE:—I am familiar with a plant that originally cost $50,000 four years ago. It is stationed underground, out under the sidewalk and they pay $250 a month rent. This plant costs about $17,500 a year to operate and I was authorized recently to offer to sell the plant. The prospective customer would not give $20,000 for the plant and operate it himself. He prefers to pay $17,500 a year and let others operate it.

[COMMUNICATED AFTER ADJOURNMENT BY MR. CHAS. BLIZARD.]

I desire to reply to Mr. Leonard's statements concerning storage batteries in isolated plants. I was not present at the Boston meeting and base my reply upon Mr. Leonard's remarks as reported.

Mr. Leonard expresses the opinion that the operation of a battery in an isolated plant does, or should, cause an increase in the cost of engine room labor. With the exception of the Edison companies and street railway companies operating batteries on a very extensive scale, and having two or more installations, I do not know of an instance in which the installation of a battery has added one dollar to the labor item. On the contrary, batteries in many cases have effected a very material reduction in the engine room force.

The modern storage battery in an isolated plant does not require the attention of a so-called expert, it needs only such intelligent care as can readily be given it by a competent engineer or his assistants. Seventy-five per cent of the plants in New York City are being operated with entire success by engineers who are now in charge of their first batteries. None of the more important electrical or steam apparatus in large isolated plants receives or requires so little of the engineer's time as the battery. It is safe to state that the time devoted to successfully operated batteries in isolated plants does not exceed an average of an hour per day per battery.

Instructions covering the proper care of a battery are simple and can be followed by any man who is competent to give intelligent care to a dynamo and engine.

Concerning the justice of charging against a battery a rental for the space it occupies, it has not been my experience that a battery is installed in a part of the building which could be rented. Many batteries have been installed recently in buildings which have been in service for a number of years, and the fact that spaces for the plants were found without the necessity of dispossessing tenants would seem to indicate that the portions of the buildings given over to the batteries were not available for rental purposes.

The depreciation of a properly constructed storage battery is a very small item. As renewals of the plates become necessary they may be made with plates of the most modern form and construction. The battery is thus kept in step with the latest improvements and never becomes out of date apparatus.

New York, August 7, 1899.

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